PAPERS ON TIME AND TENSE

BY

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PREFACE

THE first eight of these papers are of a comparatively discursive and non-symbolic sort, and develop in various ways, and with various problems immediately in view, a certain way of looking at time and temporal reference. There follow five papers of a more formal and technical character, in which the philosophical attitudes expressed in some of the first eight are at once presupposed and made more precise. There is inevitably some community of content between all of these and my two books *Time and Modality* and *Past, Present, and Future*, but I have tried to select items which will in some way extend or clarify what is said in those books, and I have excluded papers whose content has now been wholly absorbed in them. I have also omitted items whose content has been taken up in the papers that have been included, though I shall mention below what these items are.

Of the eight 'philosophical' papers, the first two are concerned with the nature of temporal succession in the most general sense. The first, 'Changes in Events and Changes in Things', was originally published by the University of Kansas, having been delivered as the Lindley Lecture in that university in 1962. Much of its contents had already been given in a course of W.E.A. lectures in York towards the end of the preceding year, and one of its central ideas had been earlier developed in a short discussion note entitled 'Time after Time' which appeared in *Mind*, vol. 67, no. 266 (April 1958), pp. 244–6. In this paper (on 'Changes in Events and Changes in Things') I exploit a certain notion of propositions, facts, and events as 'logical constructions' which I have elsewhere developed in connexion with indirect speech and belief contexts, notably in the papers 'Berkeley in Logical Form', *Theoria*, vol. 21, nos. 2–3 (1955), pp. 117–22, and 'Oratio Obliqua', *Proceedings of the Aristotelian Society*, Supplementary Volume 37 (1963), pp. 115–26.

The parallel between tense-formation and *oratio obliqua* is also drawn in the second paper included here, 'On Spurious Egocentricity'. This was first published in *Philosophy* for October

1967, and was first delivered at the University of Keele in 1960, apart from its last section, which arises out of the discussion which took place there. Both this and the preceding paper are closely related to Chapter I of *Past, Present, and Future.*

There follow four papers on the general topic of time and determinism. The first of these, 'The Formalities of Omniscience', originally appeared in *Philosophy* for April 1962, having been read a year or two before that at a philosophical conference at Spode House in Staffordshire. It has obvious connexions with Chapter VII of *Past, Present, and Future.* I place it first in this group because it reflects the general point of view of 'Changes in Events and Changes in Things' rather more immediately than do the two which follow it, though these were delivered earlier. The paper on 'Contemplation and Action', which has not been previously published, was given at an East–West philosophical 'working party' at University House, Canberra, in 1957 (the proceedings of which, including the discussion of this paper, are informally reported in the *Australian Journal of Philosophy* for May 1958; I include part of this discussion). The one after it, on 'The Consequences of Actions', was part of a symposium on this subject at the Joint Session of the Mind Association and the Aristotelian Society at Aberystwyth in 1956, and was published in the thirtieth Supplementary Volume of the *Proceedings of the Aristotelian Society.* The fourth paper in this group, on 'Limited Indeterminism', is not concerned (as the preceding three are) with fore-knowledge or with deliberation, but with a rather more metaphysical aspect of the determinism-indeterminism controversy, and has close links with the two papers that come after it. It was first delivered at the conference of the Mid-Western Division of the American Philosophical Association at Detroit in 1962, and was published in the *Review of Metaphysics*, vol. 16, no. 1 (September 1962), pp. 55–61.

The next two papers are on the general topic of time, existence, and identity, discussed in Chapter VIII of *Past, Present, and Future.* The first, on 'Identifiable Individuals', appeared in the *Review of Metaphysics*, vol. 13, no. 4 (June 1960), pp. 684–96. The puzzle about creation out of nothing which forms one of its central topics had been earlier discussed in 'Creation in Science and Theology', in the astronomical journal *Southern Stars*, vol. 18, no. 4 (December 1959), and the notion of 'logical

illusion', which is brought in incidentally, is more fully discussed in 'On a Family of Paradoxes', *Notre Dame Journal of Formal Logic*, vol. 2, no. 1 (1961), pp. 16–32.

The second paper in this group, entitled 'Time, Existence, and Identity', appeared in the *Proceedings of the Aristotelian Society* for 1965–6, pp. 183–92, and concerns a problem ('Can one thing become two?') which I had discussed earlier in 'Opposite Number', *Review of Metaphysics*, vol. 2, no. 2 (December 1957), pp. 196–201.

All of the five more formal pieces at the end begin from E. J. Lemmon's 'stratified' version of the logic of tenses, in which we first develop a calculus which does not reflect any special assumption about the earlier-later relation, and on this we superimpose further postulates which reflect whatever special assumptions about that relation we may regard as true or interesting. In Paper IX, which is the only one of this group to have been published before (it appeared in *Theoria*, vol. 33 (1967), no. 1, pp. 28–38), this procedure is tentatively extended to *metric* tense logic; in Paper X, on 'The Logic of Ending Time', it is fairly straightforwardly illustrated. This last paper expands considerably the brief remarks on ending time in Chapter IV of *Past, Present, and Future.*

Paper X also expounds in passing the view (my own, not Lemmon's, and first adumbrated in Appendix B of *Past, Present, and Future*) that the logic of the earlier-later relation is something to be embedded in tense logic rather than vice versa. This reversal is the main topic of Paper XI, on 'Tense Logic and the Logic of Earlier and Later', in which I move through a succession of calculi in which tense logic is first dominated by the theory of the earlier-later relation and then progressively 'takes over'. Paper XII, on 'Quasi-propositions and Quasi-individuals', points out that a similar reversal is formally possible, whether or not it is philosophically desirable, in other areas than tense logic. That is, there are other areas in which predicates of individuals (like the earlier-later relation between instants) can be replaced by modalities of propositions (like tenses). I do not personally advocate this procedure in these areas, but it may throw light on what some other philosophers have been after. There is not much symbolism in this paper, which might for that reason have been better placed earlier,

but it is too close in subject-matter to Paper XI to be put anywhere but immediately after it.

Paper XIII detects substantial and questionable metaphysical assumptions even in Lemmon's 'non-committal' basis for tense logic, and makes detailed suggestions for its revision. The philosophical motivation for this revision is connected with the problems about individuality and existence raised in Paper VII and at the end of Papers I and XII, and discussed more fully in Chapter VIII of *Past, Present, and Future.*

I wish to thank the editors of the *Australasian Journal of Philosophy*, *Philosophy*, the *Proceedings of the Aristotelian Society*, the *Review of Metaphysics*, and *Theoria*, and the Philosophy Department of the University of Kansas, for permission to republish the papers originally published by them.

This collection is dedicated to the memory of Edward John Lemmon, who died in July 1966 in California, at the age of 36.

A. N. P.

Oxford, 1967

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I

CHANGES IN EVENTS AND CHANGES IN THINGS

THE basic question to which I wish to address myself in this lecture is simply the old one, does time really flow or pass? The problem, of course, is that genuine flowing or passage is something which occurs *in* time, and *takes* time to occur. If time itself flows or passes, must there not be some 'super-time' in which it does so? Again, whatever flows or passes does so at some *rate*, but a rate of flow is just the amount of movement in a given *time*, so how could there be a rate of flow of time itself? And if time does not flow at any rate, how can it flow at all?

A natural first move towards extricating ourselves from these perplexities is to admit that talk of the flow or passage of time is just a metaphor. Time may be, as Isaac Watts says, *like* an ever-rolling stream, but it isn't really and literally an ever-rolling stream. But *how* is it like an ever-rolling stream? What is the literal truth behind this metaphor? The answer to this is not, at first sight, difficult. Generally when we make such remarks as 'Time does fly, doesn't it?—why, it's already the 16th', we mean that some date or moment which we have been looking forward to as future, has ceased to be future and is now present and on its way into the past. Or more fundamentally, perhaps, some future *event* to which we have been looking forward with hope or dread is now at last occurring, and soon will have occurred, and will have occurred a longer and longer time *ago*. We might say, for example, 'Time does fly—I'm already 47'—that is, my birth is already that much past, 'and soon I shall be 48', i.e. it will be more past still. Suppose we speak about something 'becoming more past' not only when it moves from the comparatively near past to the comparatively distant past, but also when it moves from the present to the past, from the future to the present, and from the comparatively distant future to the comparatively near future. Then whatever is happening, has

happened, or will happen is all the time 'becoming more past' in this extended sense; and just this is what we mean by the flow or passage of time. And if we want to give the *rate* of this flow or passage, it is surely very simple—it takes one exactly a year to get a year older, i.e. events become more past at the rate of a year per year, an hour per hour, a second per second.

Does this remove the difficulty? It is far from obvious that it does. It's not just that an hour per hour is a queer sort of rate —*this* queerness, I think, has been exaggerated, and I shall say more about it in a minute—but the whole idea of events changing is at first sight a little strange, even if we abandon the admittedly figurative description of this change as a *movement.* By and large, to judge by the way that we ordinarily talk, it's *things* that change, and events don't change but *happen.* Chairs, tables, horses, people change—chairs get worn out and then mended, tables get dirty and then clean again, horses get tired and then refreshed, people learn things and forget them, or are happy and then miserable, active and then sleepy, and so on, and all these are changes, and chairs, tables, horses, and people are all what I mean by things as opposed to events. An accident, a coronation, a death, a prizegiving, are examples of what we'd call events, and it does seem unnatural to describe these as changing—what these do, one is inclined to say, is not to change but to happen or occur.

One of the things that make us inclined to deny that events undergo changes is that events *are* changes—to say that such and such an event has occurred is generally to say that some thing has, or some things have, changed in some way. To say, for instance, that the retirement of Sir Anthony Eden occurred in such and such a year is just to say that Sir Anthony then retired and so suffered the change or changes that retirement consists in—he had been Prime Minister, and then was not Prime Minister. Sir Anthony's retirement is or was a change concerning Sir Anthony; to say that it itself changes or has changed sounds queer because it sounds queer to talk of a change changing.

This queerness, however, is superficial. When we reflect further we realize that changes do change, especially if they go on for any length of time. (In this case we generally, though not always, call the change a *process* rather than an event, and there are other important differences between events and processes

besides the length of time they take, but these differences are not relevant to the present discussion, so I shall ignore them and discuss changes generally, events and processes alike.) Changes do change—a movement, for example, may be slow at first and then rapid, a prizegiving or a lecture may be at first dull and afterwards interesting, or vice versa, and so on. It would hardly be too much to say that modern science began when people became accustomed to the idea of changes changing, e.g. to the idea of acceleration as opposed to simple motion. I've no doubt the ordinary measure of acceleration, so many feet per second per second, sounded queer when it was first used, and I think it still sounds queer to most students when they first encounter it. Ordinary speech is still resistant to it, and indeed to the expression of anything in the nature of a comparison of a comparison. We are taught at school that 'more older', for example, is bad English, but why shouldn't I say that I am more older than my son than he is than my daughter? And if we have learned to talk of an acceleration of a foot per second per second without imagining that the second 'second' must somehow be a different kind of 'second' from the first one —without imagining that if motion takes place in ordinary time, acceleration must take place in some super-time—can we not accustom ourselves equally to a change of 'a second per second' without any such imagining?

Changes do change, then, but this does not leave everything quite simple and solved. For there's still something odd about the change that we describe figuratively as the flow or passage of time—the change from an event's being future to its being present, and from its being present to its being more and more past. For the other changes in events which I have mentioned are ones which go on in the event *while it is occurring;* for example, if a lecture gets duller or a movement faster then this is something it does *as it goes on;* but the change from past to still further past isn't one that occurs while the event is occurring, for all the time that an event is occurring it isn't past but present, in fact the presentness of an event just *is* its happening, its occurring, as opposed to its merely having happened or being merely about to happen. We might put it this way: the things that change are *existing* things, and it's while they exist that they change, e.g. it's existing men, not non-existent men, that get

tired and then pick up again; Julius Caesar, for example, isn't now getting tired and picking up again, unless the doctrine of immortality is true and he exists now as much as he ever did. And such changes as the change in the rate of movement are similarly changes that go on in events or processes while they exist, that is, while they exist in the only sense in which events and processes do exist, namely while they are occurring. But getting more and more past seems to be something an event does when it *doesn't* exist, and this seems very queer indeed.

We may retrace our steps to this point by looking at some of the literature of our subject. Professor C. D. Broad, in the second volume of his *Examination of McTaggart's Philosophy*, says that the ordinary view that an event, say the death of Queen Anne, is in the indefinitely distant future and then less and less future and then present and then goes into the more and more distant past—this ordinary story, Broad says, cannot possibly be true because it takes the death of Queen Anne to be at once a mere momentary thing and something with an indefinitely long history. We can make a first answer to this by distinguishing between the history that an event *has*, and the bit of history that it *is*. The bit of history that Queen Anne's death is, or was, is a very very short bit, but that doesn't prevent the history that it has from being indefinitely long. Queen Anne's death is part of the history of Queen Anne, and a very short part of it; what is long is not this part of the history of Queen Anne, but rather the history of this part of her history—the history of this part of her history is that first it was future, then it was present, and so on, and this can be a long history even if the bit of history that it is the history *of* is very short. There is not, therefore, the flat contradiction that Broad suggests here. There is, however, the difficulty that we generally think of the history of a thing as the sum of what it does and what happens to it *while it is there*—when it ceases to be, its history has ended—and this does make it seem odd that there should be an indefinitely long history of something which itself occupies a time which is indefinitely short.

But if there is a genuine puzzle here, it concerns what is actually going on also. For whatever goes on for any length of time —and that means: whatever goes on—will have future and past phases as well as the immediately present one; its going on is in

fact a continual passage of one phase after another from being future through being present to being past. Augustine's reflections, in the eleventh chapter of his *Confessions*, on the notion of a 'long time', are relevant here. Just when, he asks, is a long time long? Is it long when it is present, or when it is past or future? We need not, I think, attach much importance to the fact that Augustine concentrates on so abstract a thing as a 'time' or an interval; his problems can be quite easily re-stated in terms of *what goes on* over the interval; in fact he himself slips into this, and talks about his childhood, a future sunrise and so on. When, we may ask, does a process go on for a long time—while it is going on, or when it lies ahead of us, or is all over?

Augustine is at first driven to the view that it is when it is present that a time is long, for only what *is* can be long or short (paragraph 18). We can give the same answer with processes—it is when they are going on that they go on for a long time. But then, as Augustine points out, there are these phases. A hundred years is a long time, but it's not really present all at once, and even if we try to boil down the present to an hour, 'that one hour passes away in flying particles'. 'The present hath no space' (20). Augustine had apparently not heard of the 'specious present', but even if he had it would not have helped him much—most of the happenings we are interested in take longer than that. He tries out the hypotheses that the past and the future, and past and future events, in some sense after all 'are'—that there is some 'secret place' where they exist all the time, and from which they come and to which they go. If there is no such place, then where do those who foresee the future and recall the past, discern these things? 'For that which is not, cannot be seen' (22).

Well, Augustine says, he doesn't know anything about that, but one thing that he does know is that wherever 'time past and to come' may 'be', 'they are not there as future, or past, but present. For if there also they be future, they are not yet there; if there also they be past, they are no longer there. Wheresoever then is whatsoever is, it is only as present' (23). Of course there are present 'traces' or images of past things in our memories, and present signs and intentions on the basis of which we make our future forecasts (23, 24), and sometimes Augustine seems satisfied with this—past, present, and future, he says,

'do exist in some sort, in the soul, but otherwhere do I not see them' (26). But sometimes he seems far from content with this —*that which* we remember and anticipate, he says, is different from these signs, and is *not* present (23, 24)—and, one must surely add, is *not* 'in the soul'.

It is time now to be constructive, and as a preparation for this I shall indulge in what may seem a digression, on the subject of Grammar. English philosophers who visit the United States are always asked sooner or later whether they are 'analysts'. I'm not at all sure what the answer is in my own case, but there's another word that Professor Passmore once invented to describe some English philosophers who are often called 'analysts', namely the word 'grammaticist', and that's something I wouldn't at all mind calling myself. I don't deny that there are genuine metaphysical problems, but I think you have to talk about grammar at least a little bit in order to solve most of them. And in particular, I would want to maintain that most of the present group of problems about time and change, though not quite all of them, arise from the fact that many expressions which look like nouns, i.e. names of objects, are not really nouns at all but concealed verbs, and many expressions which look like verbs are not really verbs but concealed conjunctions and adverbs. That is a slight over-simplification, but before we can get it stated more accurately we must look more closely at verbs, conjunctions, and adverbs.

I shall assume that we are sufficiently clear for our present purposes as to what a noun or name is, and what a sentence is; and given these notions, we can define a verb or verb-phrase as an expression that constructs a sentence out of a name or names. For instance, if you tack the verb 'died' on the name 'Queen Anne' you get the sentence ' Queen Anne died', and if you tack the phrase 'is an undertaker' on the name 'James Bowels' you get the sentence 'James Bowels is an undertaker', so that this is a verb-phrase. I say 'out of a name *or names*' because some verbs have to have an object as well as a subject. Thus if you put the verb 'loves' between the names 'Richard' and 'Joan' you get the sentence 'Richard loves Joan'; this verb constructs this sentence out of these two names; and the phrase 'is taller than' would function similarly. Logicians call verbs and verb-phrases 'predicates'; 'died' and 'is an undertaker' would be 'one-place'

predicates, and 'loves' and 'is taller than' are 'two-place' predicates. There are also expressions which construct sentences, not out of names, but out of other sentences. If an expression constructs a sentence out of two or more other sentences it is a conjunction, or a phrase equivalent to a conjunction. For example 'Either—or—' functions in this way in 'Either it will rain or it will snow'. If the expression constructs a sentence out of one other sentence it is an adverb or adverbial phrase, like 'not' or 'It is not the case that', or 'allegedly' or 'It is alleged that', or 'possibly' or 'It is possible that'. Thus by attaching these expressions to 'It is raining' we obtain the sentences

It is not raining;
It is not the case that it is raining;
It is allegedly raining;
It is alleged that it is raining;
It is possibly raining;
It is possible that it is raining.

One very important difference between conjunctions and adverbs, on the one hand, and verbs, on the other, is that because the former construct sentences out of sentences, i.e. the same sort of thing as they end up with, they can be applied again and again to build up more and more complicated sentences, like 'It is allegedly possible that he will not come', which could be spread out as

It is said that (it is possible that (it is not the case that (he will come))).

You can also use the same adverb twice and obtain such things as double negation, alleged allegations and so on. Verbs, because they do not end up with the same sort of expression as what they start with, cannot be piled up in this way. Having constructed 'Queen Anne died' by the verb 'died' out of the name 'Queen Anne', you cannot do it again—'Queen Anne died died' is not a sentence.

Turning now to our main subject, I want to suggest that putting a verb into the past or future tense is exactly the same sort of thing as adding an adverb to the sentence. 'I *was* having my breakfast' is related to 'I am having my breakfast' in exactly the same way as 'I am *allegedly* having my breakfast' is related

to it, and it is only an historical accident that we generally form the past tense by modifying the present tense, e.g. by changing 'am' to 'was', rather than by tacking on an adverb. In a rationalized language with uniform constructions for similar functions we could form the past tense by prefixing to a given sentence the phrase 'It was the case that', or 'It has been the case that' (depending on what sort of past we meant), and the future tense by prefixing 'It will be the case that'. For example, instead of 'I will be eating my breakfast' we could say

> 'It will be the case that I am eating my breakfast,'

and instead of 'I was eating my breakfast' we could say

> 'It was the case that I am eating my breakfast'.

The nearest we get to the latter in ordinary English is 'It was the case that I *was* eating my breakfast', but this is one of those anomalies like emphatic double negation. The construction I am sketching embodies the truth behind Augustine's suggestion of the 'secret place' where past and future times 'are', and his insistence that wherever they are, they are not there as past or future but as present. The past is not the present but it *is* the past present, and the future is not the present but it *is* the future present.

There is also, of course, the past future and the future past. For these adverbial phrases, like other adverbial phrases, can be applied repeatedly—the sentences to which they are attached do not have to be simple ones; it is enough that they be sentences, and they can be sentences which already have tense-adverbs, as we might call them, within them. Hence we can have such a construction as

> 'It will be the case that (it has been the case that (I am taking off my coat))',

or in plain English, 'I will have taken off my coat'. We can similarly apply repeatedly such *specific* tense-adverbs as 'It was the case forty-eight years ago that'. For example, we could have

> 'It will be the case seven months hence that (it was the case forty-eight years ago that (I am being born))',

that is, it will be my forty-eighth birthday in seven months' time.

To say that a change has occurred is to say at least this much: that something which was the case formerly is not the case now. That is, it is at least to say that for some sentence *p* we have

> It was the case that *p*, and it is not the case that *p*.

This sentence *p* can be as complicated as you like, and can itself contain tense-adverbs, so that one example of our formula would be

> It was the case 5 months ago that (it was the case only 47 years ago that (I am being born)), and it is not now the case that (it was the case only 47 years ago that (I am being born)),

that is, I am not as young as I used to be. This last change, of course, is a case of precisely that recession of events into the past that we are really talking about when we say that time flows or passes, and the piling of time-references on top of one another, with no suggestion that the time-words must be used in a different sense at each level, simply reflects the fact that tense-adverbs *are* adverbs, not verbs.

An important point to notice now is that while *I* have been talking about words—for example about verbs and adverbs—for quite a long time, the sentences that I have been using as examples have *not* been about words but about real things. When a sentence is formed out of another sentence or other sentences by means of an adverb or conjunction, it is not *about* those other sentences, but about whatever they are themselves about. For example, the compound sentence 'Either I will wear my cap or I will wear my beret' is not about the sentences 'I will wear my cap' and 'I will wear my beret'; like them, it is about me and my headgear, though the information it conveys about these is a little less definite than what either of them would convey separately. Similarly, the sentence 'It will be the case that I am having my tooth out' is not about the sentence 'I am having my tooth out'; it is about me. A genuine sentence about the sentence 'I am having my tooth out' would be one stating that it contained six words and nineteen letters, but 'It will be the case that I am having my tooth out', i.e. 'I will be having my tooth out', is quite obviously not a sentence of this sort at all.

Nor is it about some abstract entity named by the clause 'that I am having my tooth out'. It is about me and my tooth, and about nothing else whatever. The fact is that it is difficult for the human mind to get beyond the simple subject-predicate or noun-verb structure, and when a sentence or thought hasn't that structure but a more complex one we try in various ways to force it into the subject-predicate pattern. We thus invent new modes of speech in which the subordinate sentences are replaced by noun-phrases and the conjunctions or adverbs by verbs or verb-phrases. For example, instead of saying

(1) *If* you have oranges in your larder you have been to the greengrocer's,

we may say

(2) Your having oranges in your larder *implies* your having been to the greengrocer's,

which looks as if it has the same form as 'Richard loves Joan' except that 'Your having oranges in your larder' and 'Your having been to the grocer' seem to name more abstract objects than Richard and Joan, and implying seems a more abstract activity than loving. We can rid ourselves of this suggestion if we reflect that (2) is nothing more than a paraphrase of (1). Similarly,

(3) It is now six years since it was the case that I am falling out of a punt,

could be re-written as

(4) My falling out of a punt has receded six years into the past.

This suggests that something called an event, my falling out of a punt, has gone through a performance called receding into the past, and moreover has been going through this performance even after it has ceased to exist, i.e. after it has stopped happening. But of course (4) is just a paraphrase of (3), and like (3) is not about any objects except me and that punt—there is no real reason to believe in the existence either now or six years ago of a further object called 'my falling out of a punt'.

What I am suggesting is that what looks like talk about events is really at bottom talk about things, and that what looks

like talk about changes in events is really just slightly more complicated talk about changes in things. This applies *both* to the changes that we say occur in events when they are going on, like the change in speed of a movement ('movement' is a *façon de parler;* there is just the moving car, which moves more quickly than it did), *and* the changes that we say occur in events when they are not going on any longer, or not yet, e.g. my birth's receding into the past ('birth' is a *façon de parler*—there's just me being born, and then getting older).

It's not all quite as simple as this, however. This story works very well for me and my birth and my fall out of the punt, but what about Queen Anne? Does Queen Anne's death getting more past mean that *Queen Anne* has changed from having died 250 years ago to having died 251 years ago, or whatever the period is?—that *she* is still 'getting older', though in a slightly extended sense? The trouble with this, of course, is just that Queen Anne doesn't exist now any more than her death does. There are at least two different ways in which we might deal with this one. We might, in the first place, say that our statement really is about Queen Anne (despite the fact that she 'is no more'), and really is, or at least entails, a statement of the form

> It was the case that *p*, and is not now the case that *p*,

namely

> It was the case that it was the case only 250 years ago that Queen Anne is dying, and is not now the case that it was the case only 250 years ago that Queen Anne is dying,

but we may add that this statement does not record a 'change' in any natural sense of that word, and certainly not a change in Queen Anne. A genuine record of change, we could say, must not only be of the form above indicated but must meet certain further conditions which we might specify in various ways. And we could say that although what is here recorded *isn't* a change in the proper sense, it is *like* a change in fitting the above formula. The flow of time, we would then say, is merely metaphorical, not only because what is meant by it isn't a genuine movement, but further because what is meant by it isn't a genuine change; but the force of the metaphor can still be explained—we use the metaphor because what we call the flow of time

does fit the above formula. On this view it might be that not only the recession of Queen Anne's death but my own growing older will not count as a change in the strict sense, though growing older is normally *accompanied* by genuine changes, and the phrase is commonly extended to cover these—increasing wisdom, bald patches, and so on.

But can a statement really be *about* Queen Anne after she has ceased to be? I do not wish to dogmatize about this, but an alternative solution is worth mentioning. We might paraphrase 'Queen Anne has died' as 'Once there was a person named 'Anne', who reigned over England, etc., but there is not now any such person'. This solution exploits a distinction which we may describe as one between *general facts* and *individual facts*. That someone has stolen my pencil is a general fact; that John Jones has stolen my pencil, if it is a fact at all, is an individual fact. It has often been said—for example, it was said by the Stoic logicians—that there are no general facts without there being the corresponding individual facts. It cannot, for example be the case that 'someone' has stolen my pencil, unless it is the case that some specific individual—if not John Jones, then somebody else—has stolen it. And in cases of this sort the principle is very plausible, indeed it is obviously true. I have read that some of the schoolmen described the subject of sentences like 'someone has stolen my pencil' as an *individuum vagum*, but of course this is a makeshift—forcing things into a pattern again. There are no 'vague individuals', and if a pencil has been stolen at all it has been stolen not by a vague individual but by some quite definite one, or else by a number of such. There are vague statements, however, and vague thoughts, and the existence of such statements and thoughts is as much a fact about the real world as any other; and when we describe the making of such statements and the entertaining of such thoughts, we do encounter at least partly general facts to which no wholly individual facts correspond. If I allege or believe that someone has stolen my pencil, there may be *no* specific individual with respect to whom I allege or believe that *he* stole my pencil. There is *alleged or believed to be* an individual who stole it, but there is *no individual who is alleged or believed* to have stolen it (not even a vague one). So while it is a fact that I allege or believe that someone stole it, there is no fact of the

form 'I allege (or believe) that *X* stole it'. The one fact that there is, is no doubt an individual fact in so far as it concerns me, but is irreducibly general as far as the thief is concerned. (There may indeed be *no* thief—I am perhaps mistaken about the whole thing—but this is another question; our present point is that there may be no one who is even said or thought to be a thief, though it is said or thought *that there is* a thief.)

Returning now to Queen Anne, what I am suggesting is that the sort of thing that we unquestionably do have with 'It is said that' and 'It is thought that', we also have with 'It will be the case that' and 'It was the case that'. It *was the case that someone* was called 'Anne', reigned over England, etc., even though *there is not now anyone* of whom it was the case that *she* was called 'Anne', reigned over England, etc. What we must be careful about here is simply getting our prefixes in the right order. Just as

(1) I think that (for some specific *X* (*X* stole my pencil))

does not imply

(2) For some specific *X* (I think that (*X* stole my pencil)),

so

(3) It was the case that (for some specific *X* (*X* is called 'Anne', reigns over England, etc.))

does not imply

(4) For some specific *X* (it was the case that(X is called 'Anne', reigns over England, etc.)).

On this view, the fact that Queen Anne has been dead for some years is not, in the strict sense of 'about', a fact about Queen Anne; it is not a fact about anyone or anything—it is a *general* fact. Or if it is about anything, what it is about is not Queen Anne—it is about the earth, maybe, which has rolled around the sun so many times since there was a person who was called 'Anne', reigned over England, etc. (It would then be a *partly* general fact—individual in so far as it concerns the earth, but irreducibly general as far as the dead queen is concerned. But if there are—as there undoubtedly are—irreducibly partly general facts, could there not be irreducibly wholly general

ones?) Note, too, that the fact that this fact is not about Queen Anne, cannot itself be a fact about Queen Anne—its statement needs rephrasing in some such way as 'There is no person who was called 'Anne', etc., and about whom it is a fact that, etc.'

On this view, the recession of Queen Anne's death into the further past is quite decidedly not a change in Queen Anne, not because we are using 'change' in so tight a sense that it is not a change at all, but because Queen Anne doesn't herself enter into this recession, or indeed, now, into any fact whatever. But the recession *is* still a change or quasi-change in the sense that it fits the formula 'It was the case that *p*, but is not now the case that *p*'—this formula continues to express what is common to the flow of a literal river on the one hand (where it was the case that such-and-such drops were at a certain place, and this is the case no longer) and the flow of time on the other.

II

ON SPURIOUS EGOCENTRICITY

1. It is frequently said that words like 'now', 'then', 'ago', 'present', 'past', 'future', and the various indications of tense, are 'egocentric' or 'token-reflexive' in character. I want to suggest, on the contrary, that the apparent egocentricity or token-reflexiveness of this class of expression is deceptive. It is perhaps not easy to see how on a point of this sort deception is possible, but a parallel case may make the position clearer.

2. The 108th of the 'Miscellaneous Examples for the Exercise of Learners' appended to Whately's *Elements of Logic* runs as follows: 'He who believes himself to be always in the right in his opinion, lays claim to infallibility: you always believe yourself to be in the right in your opinion: therefore you lay claim to infallibility.' What we are intended to say about this is clearly that as a syllogism it is invalid through having four terms, 'believing oneself to be always in the right' (in the major) being a different thing from 'always believing oneself to be in the right' (in the minor); but the fact that these are different is itself a fact of some significance. It is not possible to believe anything seriously without believing that this believed thing is the case, or is true; nor can one person *A* sincerely agree with another person *B* on any matter, or sincerely pronounce *B*'s opinions to be true or right in this matter, unless *B*'s opinions coincide with his own, or rather unless he takes *B*'s opinions to coincide with his own—this sharing of *B*'s opinions or supposed opinions is a *sine qua non* of sincerity on this point, i.e. of not only saying but thinking and believing that what *B* believes is the case or is true. To that extent the phrases 'It is true that—' and 'It is the case that—' could well be described as 'egocentric' expressions. Yet it seems perfectly clear that this implicit reference to the opinions of the speaker is an 'inseparable accident' of the use of these phrases rather than part and parcel of their meaning. 'It

is the case that *p*' just does not mean 'It is my opinion that *p*'; and 'You are right in thinking that *p*' just does not mean 'It is not only your opinion that *p*, but mine also'. For a man can unquestionably use the expression 'It is the case' and 'You are right' sincerely and seriously without for a minute imagining that he himself is always right and never wrong in his opinions (as he would have to if he thought that 'being right' simply meant 'being believed by him'). In the terms of Whately's example, 'always believing oneself to be in the right' (i.e. in what one believes at the time) is inevitable, but if this were because 'in the right' meant 'believed by me', then 'believing oneself to be always in the right' would be equally inevitable; but it isn't.

It has been often observed in this connexion that although there is a certain absurdity in a person's saying 'I believe that grass is green but of course it isn't really', there is no absurdity in another person's saying about the first one '*He* believes that grass is green but it isn't really'. This difference between 'I' and 'he' in a way strengthens the temptation to equate the meaning of 'true' with that of 'believed by me', and of 'really' with 'in my opinion'. For if we think of it this way, the difference is easily explainable—'I believe that grass is green but it isn't really so' reduces to the simple contradiction 'I believe it is and I don't believe it is', while 'He believes that it is but it isn't really' reduces to 'He believes that it is but I don't', where the contradiction disappears through the two parts of the conjunction having different subjects. But of course this explanation won't do—'I believe that grass is green but it isn't really' isn't a simple contradiction because it could quite easily happen to be true (it would be if the observer's 'he' statement were).

There is, too, a three-valuedness about 'believed by me' that there isn't about 'true'. Just as I cannot sincerely pronounce a thing true, or agree with it, without believing it, so I cannot sincerely pronounce it false, or disagree with it, unless I disbelieve it. But what is not believed by me is not always disbelieved by me—I might have no opinion about it at all, might in fact never have thought about it. What is not true, however, is false (at least where it makes sense to talk of 'truth' at all).

An egocentric theory of truth, based on the facts adduced above, would surely be a simple *hysteron proteron*. I think true

only those opinions with which I agree, not because this agreement is what 'truth' means, but on the contrary because 'agreeing with *X*' means thinking true what *X* thinks true. And the absurdity of 'I believe that grass is green but it isn't really' lies not in simple self-contradiction but in the fact that such a statement can only be true if its utterance is insincere, meaning by an 'insincere' utterance one in which the speaker asserts either something that he does not believe or the contradictory of something that he does believe. If, in the given case, he really does believe that grass is green (so that the first clause of his utterance is true), and it really isn't (so that the second clause is also true), he is insincere because in saying (truly) that it isn't he contradicts his (admitted and genuine) belief that it is. The outsider, of course, can report the two facts without either falsehood or insincerity.

3. There is another theory of truth which is sometimes confused with the egocentric theory, and does share with it an appearance of mild scepticism, but which is in fact totally different. I mean what is sometimes called the 'no-truth' theory of Ramsey and Ayer—the theory that 'It is true that grass is green', 'It is the case that grass is green', 'Grass is really green', all just amount to 'Grass is green'. There is certainly at least complete mutual entailment between these forms, and interchangeability without change of truth-value in most contexts that I can think of, including *oratio obliqua*—there is not only no difference in logical force between the propositions just cited, but none, either, between 'It is true that he believes that grass is green', 'He believes that grass is green', and 'He believes that it is true that grass is green'. The allegedly equivalent 'I believe that' will not preserve its vacuity in these contexts—'I believe that he believes that grass is green' need not have the same truth-value as the plain 'He believes that grass is green', and still more certainly neither of these need have the same truth-value as 'He believes that I believe that grass is green'. (I say 'still more certainly' because I cannot at the time of saying them discriminate both sincerely and with truth between the first two statements, I mean distinguish them as the one true and the other false, but I can certainly thus discriminate between either of them and the third.)

One might put the 'no-truth' theory in a technical context thus: in certain symbolic systems a variable, say '*d*', is used to stand indifferently for any expression which constructs a statement out of a statement, e.g. 'It is not the case that—', 'Grass is green and—', 'If—then grass is green', 'It is possible that—', 'Aristotle asserted that—'. Thus '*d* (The sky is blue)' can stand indifferently for 'It is not the case that the sky is blue', 'Grass is green and the sky is blue', 'If the sky is blue then grass is green', and so on. In expressing logical generalizations by means of such variables it is useful to include 'The sky is blue' itself, without any adornments, as being among the things that '*d* (The sky is blue)' can stand for. But here there is no actual expression that has been put in the place of '*d*', and we tend to feel that this is awkward. The awkwardness, if we are really worried about it, can be relieved by simply introducing an actual expression which when put for '*d*' in '*d* (The sky is blue)' will yield a sentence meaning no more and no less than the plain 'The sky is blue'. If we used 'It is the case that—' or 'It is true that—' or 'really' or 'truly' in this way, this would be precisely its commonest use in ordinary English.

4. There is, indeed, one common sense of 'truly' that does add something to what is said, namely when it means 'correctly', as when we say that someone believes (or says) truly (or with truth) that grass is green. This certainly says more than merely that the man believes or says the thing. But the difference doesn't refute the 'no-truth' theory, since we can explain what 'truly' adds here, and eliminate the word itself, by translating the remark as 'He believes that grass is green, and it is' (and similarly with 'says'). Just as 'Truly *p*' or 'It is true (the case) that *p*' seems a mere inflation of the plain '*p*', so '*X* believes (says) truly that *p*' seems to mean no more and no less than '*X* believes (says) that *p*, and *p*'. And the problem of eliminating 'true' from statements like 'Everything that John believes is true', i.e. 'Whatever John believes he believes truly (correctly)', seems completely solved by translating it in Ramsey's manner as 'For all *p*, if John believes that *p* then *p*'.

It is perhaps less misleading to describe this 'no-truth' theory as the 'adverbial' theory, for it proceeds basically by giving a simple account of 'truly' (in the two related senses that it has

in 'Truly *p*' and in 'He says truly that *p*'), and assumes that locutions in which 'true' appears as an adjective are dispensable variations of forms in which it occurs only as part of an adverb or adverbial phrase. It is a 'pseudo-adjective' that can be attached with sense only to 'pseudo-nouns', namely 'that'-clauses and their equivalents, and to 'pseudo-descriptions' like 'what *X* thinks'. It has indeed been given a use, as by Tarski, as a genuine adjective attaching to genuine nouns which designate sentences, i.e. physical inscriptions; but this use is a highly artificial one, and need not be considered here. And it is worth observing that Tarski himself, in addition to his definitions of 'true sentence', has given some attention to a function *as*(*p*) which has precisely the properties of Ramsey's 'It is true that *p*' (*Logic, Semantics and Metamathematics*, p. 3).

5. At almost all these points the parallel with 'now' and 'present' seems as complete as any such parallel can be. Certainly if I say that some process or other is 'present' or that it is 'going on now', I will be right if and only if it is going on contemporaneously with my utterance. That is, what is present when I say it is, is always contemporaneous with my saying it. But this, it seems to me, is not at all because this is what 'being present' means, but on the contrary because 'contemporaneous with *X*' means 'present when *X* is present' ('going on when *X* is going on').

As an analogue of the sophism in Whately's exercise, we might construct this one: What is always contemporaneous with whatever is happening, must go on for ever; but always my utterances are contemporaneous with whatever is happening; therefore I never stop talking. It is bound to be true, in view of what 'contemporaneous' means, that whatever utterances are occurring (and indeed whatever events of any sort are occurring) are occurring contemporaneously with whatever is occurring. And not only is this true now but it always has been and always will be—whoever says, not only now but at any time, 'Whatever is occurring, e.g. my own talking, is contemporaneous with whatever is occurring' will say this with truth. But it by no means follows, nor is it in general true, that if I select some present occurrence *A* and say of it that at any time when any occurrence *B* was going on and was present, the

one that I have selected was going on and was present contemporaneously with *B*.

'Was going on' and 'was present'—note these phrases. The presentness of an event can itself become past, in fact automatically does so when the event in question does. And even the *material* equivalence of 'present' and 'contemporaneous with this utterance' does not survive changes of tense (just as the material equivalence, at least in the view of the believer, of 'true' and 'believed by me' does not survive embedding in *oratio obliqua*). 'His eating his breakfast is a present fact' is true when and only when 'His eating his breakfast is contemporary with this utterance' is true; but the same cannot be said of 'His eating his breakfast was a present fact' and 'His eating his breakfast was contemporaneous with this utterance' (which is only true if the utterance is one that has gone on without interruption since whenever it was that he was eating his breakfast). (Cf. our parallel to the Whately problem.)

Again, just as there must surely be many truths which no one believes (unless it be God) and no one disbelieves, since no one has thought of them, so surely things like the solidifying of the earth were future and became present and then became past—were about to happen and then happened and then had happened—without anybody saying so or being there to say so, i.e. without there being any utterances for them to be contemporaneous with or earlier or later than. It was remarked by McTaggart (*The Nature of Existence*, ch. xxxiii, § 313) that on the relative-to-utterance theory, 'if there were events earlier than any consciousness, those events would never be future or present, though they could be past', i.e. once consciousness, and utterances, had appeared (before then they couldn't be past either). This is surely, as McTaggart intended it to be, a sufficient *reductio ad absurdum* of this theory.

6. On the other hand, a 'no-present' theory analogous to the Ramsey-Ayer theory of truth has almost everything to be said for it. That is, 'He is eating his breakfast now' and 'He is eating his breakfast at present' seem to say no more and no less, apart from nuances of emphasis, than the plain 'He is eating his breakfast'. We can do without 'now', we can do without a present-tense copula 'is', we can do without even a special

present-tense inflection of the main verb—just using the root verb-form itself, as in 'I eat' and 'They eat' in English—if we understand that this is what we have with us all the time—this is what the verb form basically means.

This equivalence of the 'presentness of the occurring of *X*' with the simple 'occurring of *X*' is preserved under tense inflection, in the way that the equivalence of 'presentness' with 'contemporaneity with this utterance' (as we saw earlier) is not so preserved. This point was remarked by Thomas Aquinas, in a comment on that curious passage in Aristotle's *De Interpretatione* (16^b 17–19) in which it is said that verbs in non-present tenses are not genuine verbs at all but only 'cases' of verbs, like the oblique cases of nouns. Thomas notes that some positive connexion with genuine verbs is still suggested by Aristotle's description, and finds this connexion in the fact that the present or presentness is indirectly referred to even in talk of past or future, for it is always a present that is past or future—'est praeteritum quod fuit praesens, futurum autem quod erit praesens'. The same surely applies also on the other side of the 'tensing'; I mean, not only is pastness in all cases past presentness, but it is also in all cases present pastness, and similarly with futurity. There are, indeed, past and future pastness too, and past and future futurity; it is just this that makes it worth saying that plain pastness and futurity are present pastness and futurity ('present' is the vacuous special case of temporal modification); but the equation given in the last sentence still holds, for past and future pastness are past and future present pastness, and past and future futurity are past and future present futurity. To put it another way: The verb *tout simple* is the present-tense verb, and this is true even if the verb in question is an auxiliary verb, with other verbs depending on it, and even if, out of the various auxiliaries there are, it happens to be 'has been' or 'will be'.

Nor do these two 'vacuity' or 'omnipresence' theories—the one about 'truly' and the one about the present—constitute a mere parallelism. They are in a sense the same theory—in the sense that they merely assert the vacuity of one and the same phrase, 'It is the case that—' or 'It is true that—', considered against different non-vacuous contrasting phrases. It is, on the one hand, the vacuous member of the pair of which the non-

vacuous member is 'It is not the case that—' or 'It is false that—'. But it is also the vacuous member of a set of which other members are 'It has been the case that—' (or 'It has been true that—') and 'It will be the case that—' (or 'It will be true that—'), with variations like 'It used to be the case that—', 'It was the case *x* hours ago that—', etc. But the phrase 'It is the case (true) that—' doesn't have different senses in these two settings. It carries tense with it—the present tense—just as much when it is being contrasted with 'It is not the case (is false) that—' as when it is being contrasted with 'It has been that—' or 'It will be that—'. And conversely, it's no more than the same old 'multiply by one' business when we think of it as a tense-indication.

7. I have had, all the same, to express myself with caution at certain key points—I have said only that my parallel holds at 'almost' all points, and that the 'no-present' theory has 'almost' everything to be said for it. This has been necessary because colloquial English is not so streamlined as to fit in with the view I have sketched in all its uses of tense indicators. This admission is especially necessary with the simple 'now'. There are in fact uses of 'now' in which it really is synonymous with 'contemporaneously with this utterance', e.g. when the word is uttered in order to let someone know when to stop pouring water into one's whisky—here the actual utterance really has to be in the centre of one's consciousness. But this use of 'now' stands out so sharply from all its other uses that the very fact that the translation suits it is enough to show that it doesn't suit the others. But it is also unusual, it must be confessed, for 'now' to be directly dispensable with oblique tenses (as it ought to be by my account). We can drop it from 'He was now approaching the stairway' and from 'You will come to a lamp-post, and you will now see the house you want on your left'; and here 'was now' and 'will now' really do express past presentness and future presentness; but these, like the drinker's response to 'Say when', are rather special cases. What happens more often is that 'now' gets attached in sense to a statement as a whole, no matter how subordinate the clause in which it is immediately placed. When we say, for example, 'he said he would be in London now', we do not at all mean that he proclaimed the presentness of his

being in London; nor if we say 'You will always be proud of what you are doing now' do we mean that you will always be proud-of-what-you-are-doing. There is, I am inclined to think, a kind of periphrastic dispensability of this 'now'—the first example is equivalent to something like 'He is now due in London', and the second to 'You are now doing something on which you will always look back with pride', and the 'now' can be dropped from both of these.

This peculiarity of having the whole sentence for its 'scope' rather than the sub-sentence in which it immediately occurs, is a peculiarity which the colloquial 'now' shares with the colloquial 'any'. What the word 'any' conveys is universal quantification, and in general it has the same logical force as 'every'—'*Any* man who looks at this will find it hard to see' is equivalent to '*Every* man who looks at this will find it hard to see'; and similarly in most cases. But not in all. 'They didn't eat any apples' is distinctly stronger in force than 'They didn't eat every apple', not equivalent to it, and similarly with 'If *any* man moves I'll shoot' and 'If *every* man moves I'll shoot'. The thing about 'any' is that it gives universal quantity to the *whole sentence* in which it occurs, even if this means that sometimes it doesn't give universal quantity to its own immediate sub-sentence, as 'every' does. It must just be admitted, I think, that this deplorable holism does infect the ordinary English use of certain words, and 'any' and 'now' are among them.

8. It may, however, be argued that I have now cut the ground from under my own feet at another point. I observed earlier that even if there might be a sort of 'illocutionary inconsistency' in my saying 'He believes that grass is green' but refusing to say 'I believe that he believes that grass is green', there would be none in my saying both of these things but refusing to say 'He believes that I believe that grass is green'. (This was supposed to show the non-equivalence of 'It is true that *p*' and 'I believe that *p*'.) One might counter this by saying that here again we are confronted merely by the 'deplorable holism' which infects the English language; it is the same with 'I' as it is with 'now' and with 'any'—even in subordinate clauses we make it refer to the speaker of the entire sentence. In a more rational language we might have used a pronoun 'Self' in such a way that 'Self is

sick', for example, means the same as our present 'I am sick', but 'He believes that self is sick' would mean, not the same as our present 'He believes that I am sick', but the same as our present 'He believes that he is sick'. And then, perhaps, 'He believes that self believes that grass is green' would be related to 'He believes that it is true that grass is green' exactly as 'It is true that grass is green' is to 'I believe (i.e. Self believes) that grass is green'. Certainly 'He *says* that it is true that grass is green' would be related in this way to 'He says that self believes that grass is green'.

I do not think I want to quarrel with this (it is a point which was first made to me by Mr. J. Wiredu), and I am happy to throw away whatever additional force I may have stolen for my main contentions by the use of this example. The question still remains: what *is* the relation between 'It is true that *p*' and 'Self believes that *p*', whether these forms occur on their own or in subordinate clauses; and the correct answer still seems to me the one that I have given. It remains true, for instance, that the man who says 'Self believes that grass is green, but in fact it is not' (whether this man be *my*self or another), is not guilty of actual inconsistency, i.e. it is not impossible that both parts of his statement should be true, but only that both parts of it should be both true and sincere. It remains true that the prefix 'Self believes that—' is not vacuous in the way that 'It is true that—' is. For example, the following remark may be perfectly in order:

> 'He says that self believes that *p*, and that nevertheless it is not the case that *p;* and as it happens—though of course the poor twit doesn't know it—he's right both times';

but surely not the following:

> 'He says that *p*, and that nevertheless it is not the case that *p;* and as it happens, etc., he's right both times',

though had 'it is true that—' replaced 'self believes that—' in the first sentence, they *would* have been on the same footing.

The proposed logic of 'self', it is worth adding, would have its own difficulties. For example,

> (1) He says rightly that self believes that *p*

would not entail

(2) *Self* believes that *p*,

but rather

(3) *He* believes that *p*,

since (2) would mean what is now meant by '*I* believe that *p*', which does not follow from (1) on the proposed interpretation. Somewhat similarly, if the present tense is used in formal tense calculi in the way that I propose,

(4) He *said* rightly that it is raining

would not entail

(5) It is raining,

but rather

(6) It *was* raining.

In this case, however, we can preserve the normal rule that

(7) 'He *says* rightly that *p*' entails that *p*,

and if we read (4) as

(4) It was the case that (he says rightly that it is raining),

we can derive (6) from this by (7) and

(8) If *X* entails *Y*, then 'It was the case that *X*' entails 'It was the case that *Y*'.

Analogous moves, to explain the entailment of (3) but not (2) by (1), could possibly be worked out for a pronoun-logic, but they would be trickier.[1] In any case, as I have insisted in the previous paragraph, such possibilities do not affect my central contentions.

[1] Such moves may be carried out, for example, in the pronounless 'egocentric' logic adumbrated in Paper XII. Alternatively, we may keep closer to ordinary speech by modifying (7), as in Hector-Neri Castañeda's " 'He', A study in the logic of self-consciousness", *Ratio*, vol. 8, no. 2 (Dec. 1965), pp. 130–57).

My proposed pronoun 'self' combines the properties of Castañeda's 'I', when this is so used as to be 'inelimiable for the speaker', with those of his 'he*'.

III

THE FORMALITIES OF OMNISCIENCE

WHAT do we mean by saying that a being, God, for example, is omniscient? One way of answering this question is to translate 'God is omniscient' into some slightly more formalized language than colloquial English, e.g. one with variables of a number of different types, including variables replaceable by statements, and quantifiers binding these. In such a language, the common form of the statements

God knows that $2+2 = 4$,
God knows that $2+2 = 5$,
God knows that God knows that $2+2 = 5$

can clearly be given as

God knows that p,

and this is going to be useful in answering our opening question.

If there is no God, it would seem that all statements of the form just given are false; but even if there is a God, it would seem that some of them are. For example, God doesn't know that 2 and 2 are 5, for the simple reason that 2 and 2 *aren't* 5.

This example is sufficient to show that one way in which we might be tempted to translate 'God is omniscient' simply will not do. I mean the translation

(1) For every p, God knows that p.

At least, this won't do if we want to mean something *true* by 'God is omniscient'. For it is a general rule that we may pass from a universal proposition to any singular instantiation of it, and one instantiation of (1) would be the false proposition

God knows that $2+2 = 5$.

And we know this to be false, not by any subtle or dubious theologizing, but simply by the logic of knowledge as such—if anyone thinks that $2+2 = 5$, his state of mind isn't knowledge but error.

Still, the correction required to this first effort seems simple and obvious. When we say that anyone knows everything, it is surely understood that what we mean is that he knows everything that's true. So, as a second attempted translation of 'God is omniscient' let us put

For every p, if it is *true* that p then God knows that p,

or more simply

(2) For every p, if p then God knows that p.

And it does seem that all the instantiations of *this* proposition are things which a believer in God's omniscience *would* wish to maintain. For example

If $2+2 = 4$ then God knows that $2+2 = 4$,
If $2+2 = 5$ then God knows that $2+2 = 5$,
If God knows everything then God knows that God knows everything.

These are 'instantiations' of (2) in the sense that they are formed by simply dropping the initial quantifier and putting some actual statement for the variable 'p' at the two places where it occurs, the same statement at each place. There are also 'instantiations' of (2) in a slightly more complicated sense, and all of these are, I think, propositions which a believer in God's omniscience would wish to maintain. These more complicated instantiations of (2) are ones like

(3) For every x, if $x+2 = 4$ then God knows that $x+2 = 4$.

It is easy to see how we get instantiations of (2) like this. In place of the two 'p's after the quantifier in (2) we put, not actual statements, but *forms* of statements still containing variables, and we replace the initial quantifier 'For every p' by another one containing whatever variables occur in the replacement for 'p'.

Here is another example of the same sort: Suppose we write '$f(x)$' for any statement about x, e.g. 'f(Plato)' for any statement about Plato. Then the proposition

(4) For every f, if f(Plato) then God knows that f(Plato)

would mean that God knows everything there is to know about Plato, and this too would be an instantiation of (2). So also would

(5) For every f and x, if $f(x)$ then God knows that $f(x)$,

which asserts in plain English that God knows everything about everything.

Again, suppose we introduce a variable *d* which stands for any expression which, attached to a statement, forms a statement, for example, 'It is not the case that—', 'Johnny believes that—', and so on. Thus a formula like '$d(2+2 = 5)$' can stand indifferently for such statements as

It is not the case that $2+2 = 5$,
Johnny believes that $2+2 = 5$,
I wish it were the case that $2+2 = 5$,

and so on; and also for such statements as

$1+1 = 3$ and $2+2 = 5$,
Either my name is Percy or $2+2 = 5$,
If my name is not Percy then $2+2 = 5$,

and, for that matter, for such statements as

If $2+2 = 5$ then $2+3 = 6$,
Either $2+2 = 5$ or my name is Percy,
That $2+2 = 5$ is less surprising than that my name is Percy,

in which the expression substituted for '*d*', and making a statement by being attached to '$2+2 = 5$', isn't attached to '$2+2 = 5$' by being *prefixed* to it but rather by being as it were wrapped around it. And among the instantiations of the proposition (2) is this:

(6) For all *d*, if $d(2+2 = 5)$ then God knows that $d(2+2 = 5)$.

This could be a way of translating something rather puzzling that some of the schoolmen used to say, namely that there *is* a sense in which God knows even false propositions;[1] for He *understands* them just as completely as He understands true propositons, and of course as part of this complete understanding He understands or knows that they are false. He knows, we might say, all truths into which the idea of 2 and 2 being 5 in any way enters, e.g. the truth that it is not the case that $2+2 = 5$, that some idiotic boy believes that $2+2=5$, and so on.

[1] See, e.g., Ockham, *Tractatus de Praedestinatione*, etc., ed. P. Boehner (Franciscan Institute, 1954), pp. 56, 101B.

But now we must raise a deeply controversial point. In this statement that for every *p*, if *p* then God knows that *p*, are we to understand this verb 'knows' as a verb in the present tense, or are we not? Many very reputable philosophers, e.g. St. Thomas Aquinas, have held that God's knowledge is in some way right outside of time, in which case presumably the verb 'knows' in our translation would have to be thought of as tenseless. I want to argue against this view, on the ground that its final effect is to restrict *what God knows* to those truths, if any, which are themselves timeless. For example, God could not, on the view I am considering, know that the 1960 final examinations at Manchester are now over; for this isn't something that He or anyone could know timelessly, because it just isn't true timelessly. It's true now, but it wasn't true a year ago (I write this on 29th August 1960) and so far as I can see all that can be said on this subject timelessly is that the finishing-date of the 1960 final examinations is an earlier one than 29th August, and this is *not* the thing we know when we know that those examinations are over. I cannot think of any better way of showing this than one I've used before, namely,[1] the argument that what we know when we know that the 1960 final examinations are over can't be just a timeless relation between dates because this isn't the thing we're *pleased* about when we're pleased that the examinations are over. In any case it seems an extraordinary way of affirming God's omniscience if a person, when asked what God knows *now*, must say 'Nothing', and when asked what He knew yesterday, must again say 'Nothing', and must yet again say 'Nothing' when asked what God will know tomorrow.

Of course if we take the 'knows' in our translation to be the ordinary present-tense 'knows', then we must regard the 'is' in the thing it's a translation *of*, namely 'God is omniscient', as the ordinary present-tense 'is', and if we want to translate the belief that God's omniscience is a permanent and unalterable thing we must expand our (2) to this:

(7) It is, always has been, and always will be the case that for all *p*, if *p* then God knows that *p*.

We may further note that even with respect to what God is

[1] A. N. Prior, 'Thank goodness that's over', *Philosophy* (Jan. 1959), p. 17. Cf. C. D. Broad, *Examination of McTaggart's Philosophy*, vol. ii, part i, pp. 266–7.

said in both formula (2) and formula (7) to know *now*, the statements over which the variable '*p*' may range include not only present-tense but past-tense and future-tense ones, and tenseless ones also, if any such there be. For example, we can infer by instantiation from both (2) and (7) the following propositions:

> If there were living organisms a million years ago, God knows that there were living organisms a million years ago;

and

> If there will be living organisms a million years hence, God knows that there will be living organisms a million years hence.

Omniscience as here defined, in other words, covers *fore*-knowledge of whatever will be, that is, *knowledge at every moment of whatever at that moment will be.*

But now I want to raise a subtler point. Is the believer in God's omniscience committed to the following proposition:

> (8) For all *p*, if (it is the case that) *p*, God has always known that it would be the case that *p*

(for example, if I now scratch my head, God has always known that I would scratch my head on this occasion)? The first thing to be said about this proposition is that it *isn't* a simple logical consequence of God's omniscience in the sense of our proposition (7). It does follow from proposition (7) that

> (9) If, at any time, it was the case at that time that it would be the case that *p*, then God knew at that time that it would be the case that *p*.

But this is not enough to give us proposition (8), unless we supplement it by

> (10) For all *p*, if (it is the case that) *p* then it has always been the case that it would be the case that *p*.

If (10) is true, and God is omniscient in the sense of proposition (7), then (8) is true. And contrariwise if (10) is false, then (8) must be false also. For if (10) is false, that means that in some cases in which it *is* the case that *p*, it nevertheless *hasn't* always been the case that *p* would be the case, and if this hasn't always

been true, then clearly neither God nor anyone can have *known* it to be true. So that at this point everything really depends on whether for every *p* that *is* the case, it has always been the case that it would come to pass that *p*, or as I sometimes loosely put it, whether whatever *is* the case *has always been going to be* the case. This proposition, together with God's omniscience, *does* yield the conclusion that with respect to whatever is now the case, God has always known that it would be the case; but that conclusion, i.e. our proposition (8), does *not* follow from God's omniscience alone. And my own view would be that, whatever may be the case with the doctrine of God's omniscience itself, proposition (8) is not true, nor is proposition (10). And on both these points, I mean the denial of the logical proposition (10) and even of the theological proposition (8), I rather think that, for what this is worth, I have St. Thomas Aquinas on my side, though this involves some very tricky questions of exposition.

Let's look, anyhow, at a bit of what Thomas has to say about these matters in his *De Veritate*, Question 2, Article 12, 'Whether God knows singular future contingents'.[1] He begins by stating some twelve arguments for the negative, of which I think the most persuasive is the seventh, which begins like this: given any true proposition of the form 'If *p* then *q*', if the antecedent *p* is absolutely necessary, then the consequent *q* must be absolutely necessary also. The point of this *necessarium absolute*, and the sort of necessity with which Thomas is contrasting it, is obvious enough. If I make a statement of the form 'If *p* then necessarily *q*', I may not mean that from the truth of *p* we can infer that *q* is in itself a *necessary truth*, i.e. I may not mean 'If *p* then necessarily-*q*'; I may only mean that the truth of *q*—it could quite well be the *contingent* truth of *q*—*necessarily follows* from the truth of *p*, i.e. I may only mean 'If *p* then-necessarily *q*', This is not an absolute but a merely conditional necessity of *q*; in fact not really a necessity of *q* at all, but only a necessary connexion between *q* and something else. Nevertheless we can legitimately infer the necessity of *q* in itself if we are given not only its necessary following from *p*, but also the necessity of *p* in itself. What necessarily follows from something necessary is itself

[1] Directly on the maxim that *id quod est verum in praesenti, semper fuit verum esse futurum*, it is worth also glancing at *Summa Theologica*, part i, question 16, article 7, objection 3 and answer.

necessary. That's the first premiss that Thomas's imaginary objector uses here, and Thomas himself, we shall find, quite explicitly assents to it.[1]

Now here, the objector goes on, is a true proposition of the form 'If p then q': 'If anything is known to God, then that thing will be'. This perhaps needs filling out a little; it is clear from the context that what the objector has in mind is any proposition of the general form 'If it has come to God's knowledge that X will happen, then X *will* happen'. But the antecedent of this, at least if it's true at all, is necessary, if only because it's *past*, and so beyond anyone's power to prevent—*quod fuit, non potest non fuisse*, 'What has been, cannot now not have been'.[2] So anything that follows from this necessary, i.e. now-unpreventable, truth, must itself be now-unpreventable. From this in turn it follows—the corollary is too obvious for Thomas to bother drawing it explicitly—that whatever *isn't* now-unpreventable *hasn't* yet come to God's knowledge. That's against proposition (8); it is clear that a similar argument could be used to show, against proposition (10), that whatever *has already come to be* part of what is to come is now-unpreventable, and so whatever isn't yet unpreventable hasn't yet come to be part of what is to come.

The general point of this type of argument might be brought out by using a few symbols. Suppose we use '$X = Y$' to assert that the propositions X and Y are logically equivalent, i.e. inferable from one another; this equivalence having the usual properties of symmetry (if $X = Y$, $Y = X$) and transitiveness (if $X = Y$ and $Y = Z$, $X = Z$) and also the property that if $X = Y$ then $f(X) = f(Y)$, where f is any logical function, e.g. 'Not'. Let us, further, include among logical functions of a proposition p the functions 'Necessarily p' (or 'Now-unpreventably p'), symbolized as 'Lp'; 'It was the case n time-units ago that p', written 'Pnp'; 'It will be the case n time-units hence that p', written 'Fnp'; and 'God knows that p', written 'Gp'. It is easy to show that if we have the logical equivalence

$$Pnp = LPnp$$

[1] Stock sources for this law in Aristotle are *An. Pr.* i, ch. 15, 34^a, 23, and *An. Post.* i, ch. 6, 75a, 1–11.

[2] The main stock source for this is the *Nicomachean Ethics* vi. 1139^b. See also *De Caelo* i, 283b 13.

('It was the case that p if and only if it now-unpreventably was the case that p') and any logical equivalence of the form

(i) $X = PnY$,

we can prove

(ii) $X = LX$

for we have $X = PnY = LPnY = LX$. Thus if we have

(iii) $p = PnGFnp$

('p is the case if and only if n time-units ago God knew that p would be the case n time-units later'; i.e. approximately proposition (8)), then we have

$$p = PnGFnp = LPnGFnp = Lp;$$

while if we have

(iv) $p = PnFnp$

('p is the case if and only if it was the case n time-units ago that p would be the case n time-units later'; i.e. approximately proposition (10)), then we have

$$p = PnFnp = LPnFnp = Lp.$$

And if we have

(v) $p = Gp$

('p if and only if God knows that p'), then we can derive (iii) from (iv) and vice versa (my earlier point about the deductive equivalence of (8) and (10), given God's omniscience); for by (v), $Fnp = GFnp$, and so $PnFnp = PnGFnp$.

I wish I knew where Thomas got this seventh objection from. It was developed very powerfully *after* Thomas by the fifteenth-century anti-Occamist Louvain philosopher Peter de Rivo;[1] and Peter de Rivo knew, but so far as I can discover Thomas did not know, Cicero's *De Fato*, in which a very similar argument is put into the mouth of Diodorus the Megarian.[2] In the absence of any better theory, I would suggest that perhaps

[1] See L. Baudry's excellent collection of texts, *La Querelle des Futurs Contingents (Louvain, 1465–75)*, Vrin, 1950; e.g. p. 70.

[2] Cicero, *De Fato* vii 14.

Thomas himself constructed this argument against the theological proposition (8) on the pattern of the analogous argument against the logical proposition (10) which he found in Aristotle's *De Interpretatione*, and enlarged upon (and so far as I can see accepted) in his commentary on that work, Book I, *Lectio* 13. But wherever Thomas got the argument from, it seems to me, with one slight modification that I'll discuss later, entirely conclusive. Thomas treats it, too, with the respect it deserves, and brushes aside three ways of dealing with it which he considers inadequate before putting forward an answer of his own.

There are some, he tells us, who argue that the antecedent of this conditional, despite its expressing a truth about the past, is contingent. For, these people say, it has a reference to the future, and that sort of truth about the past isn't always unpreventable —we do sometimes say truly that a thing was going to happen, and then when the time comes it doesn't. Thomas admits this sense of 'going to happen'—we do sometimes say that a thing was going to happen when we mean that everything was so to speak pointing that way—but even in this sense of 'going to happen', Thomas points out, if it's ever true that a thing was going to happen, then it cannot by that time not have been going to happen in that sense, even though perhaps by this time it isn't happening and it's clear that it never will.

Others, again, say that the proposition 'It has come to God's knowledge that *X* will happen' is contingent because it's a compound proposition with a contingent component, like 'Peter is a white man'—Peter cannot but be a man, but he needn't be white, and so needn't be a white man. To this Thomas replies that the necessity or contingency of a proposition doesn't depend on the character of its subject-matter but on the nature of the main 'link' in its construction; for example, 'I believe that man is an animal' and 'I believe that Peter is running' are equally contingent though the thing believed is necessary in the one case and contingent in the other. I'm not sure that this rule of Thomas's always works; with simple conjunctions like 'Peter is an animal and Peter is running' the modality of the whole *does* depend up to a point on the modality of the bits; but the point perhaps is that what we have here isn't that sort of combination but rather the sort in which there is definitely a principal clause ('It has come to God's knowledge—') and a subordinate one,

and here the pastness and consequent 'necessity' of the principal clause does seem to settle the matter.[1]

While I think Thomas was right on the main point here, I ought to mention that this is what was called in question by most of the writers subsequent to him who considered this argument and were not satisfied with his handling of it; for example, the fifteenth-century Occamists. They held that an element of futurity even in a subordinate clause could destroy the sort of necessity which normally attaches to past-tense truths, and in fact made such truths essentially future in sense even if past in form.[2] If, to construct a new example, my own future choice and nothing else can cause me to start smoking tomorrow, then my own future choice and nothing else can cause it to *have been* the case yesterday that I would start smoking in two days' time from then; and this fact that is directly about yesterday and only indirectly about tomorrow, if it is a fact at all, is as much a contingent fact as the one that is directly about tomorrow. Nor have Occamists hesitated to ascribe a like contingency to God's foreknowledge. But I must confess to a difficulty here. I think I can attach intelligible senses to the phrases 'was *true* yesterday' and 'was *the case* yesterday' which give the Occamist results; but I cannot find any such sense for 'was *known* yesterday'. I can by my free choice, not exercised until tomorrow, cause a person's *guess*, made yesterday, to have been a correct one (I do this simply by deciding to do what he guessed I would); and I can by the same act convey the same retrospective verification to another person's guess, made right now, that the first person's guess *was* a correct one. It is so to speak still open to this latter guess, despite its past-tense subject matter, either to turn out to have been correct or to turn out not to have been correct; its present correctness, if it does turn out to have been correct, is thus entirely contingent. But while contingent futures, and contingent future-infected pasts, can in this way be correctly or incorrectly guessed, I cannot see in what way they can be 'known'; or to put it another way, I cannot see in what way the alleged

[1] On these two very different ways in which a proposition may be compounded out of past-tense and future-tense elements, see Peter de Rivo in Baudry, op. cit., p. 339.

[2] See Ockham himself on this, op. cit., pp. 5–6C, and Ferdinand of Cordova in Baudry, op. cit., p. 159. For what seems to be a very similar view, see Ryle's *Dilemmas*, 'It Was to Be'.

knowledge, even if it were God's, could be more than correct guessing. For there would be *ex hypothesi* nothing that could *make* it knowledge, no present *ground* for the guess's correctness which a specially penetrating person might perceive.[1] So if we talk this way, while we do get my proposition (10) in a rather trivial way, I don't think we get my proposition (8), because I don't think we get the thing that ties the two together, namely God's omniscience, except in the weak sense that He *knows whatever is knowable*, this being no longer coextensive with what is true. This conclusion (that you don't get omniscience this way) seems confirmed by the fact that Ockham, who I suppose was the classical exponent of the point of view I've just been sketching, was driven to assert that 'it is impossible to express clearly the manner in which God knows future contingencies'.[2]

Returning now to Thomas: he goes on to consider a third way of answering this 'seventh objection', namely by arguing that a necessary antecedent of a true conditional *can* have a contingent consequent, as in, for example, 'If the sun shines this tree will flower'. The sun cannot but shine; but something *could* interfere with its influence on the tree so that it doesn't flower after all. Only where the connexion between antecedent and consequent is immediate, without the possibility of anything intervening to frustrate it, does the rule really hold. So it is argued; but Thomas argues on the contrary that it is only where there is no possibility of frustration that the conditional is strictly true. 'If the sun shines the tree will flower' is for this reason *not* true; if it *were* true, the necessity of its antecedent *would* be conveyed to its consequent.

Once again I think Thomas is right, but there is something that ought to be added here to obviate a misunderstanding. Some writers on these topics have thought it important to insist that no sort of knowledge, not knowledge of what is to come any more than knowledge of what has been, actually *causes* the truth of that which is known. Thomas doesn't insist on this, I think because in the case of divine knowledge he doesn't believe it is

[1] I owe much in this paragraph to Professor J. M. Shorter. Cf. also, on the negative point, Jonathan Edwards on the Will, Part II, Section xii, Observation ii. But this is a frequently repeated Thomist point too—that there can logically be no *knowledge* of the future, for one who is still awaiting its actualization, but what he can gather from its already present causes. (See, e.g., *De Malo* xvi 7.)

[2] Ockham, op. cit., p. 15.

true. Personally I do think it is true, but not very relevant to the argument we are considering. For a conditional proposition such as 'If it has come to God's knowledge that *X* will be, then *X* will be', doesn't require for its truth, or for its conveying necessity from its antecedent to its consequent, that its antecedent should *causally bring about* its consequent. It is enough that the former cannot be the case without the latter being the case, regardless of why this is so. And in fact if we like to say that it is because *X* will be that it can be known that it will be, rather than vice versa, this means more than ever that *X*'s future coming to pass is beyond prevention, since it has already *had consequences* which its opposite could not have (I take this point from Jonathan Edwards,[1] who reproduced this Objection 7 in the eighteenth century for a different purpose—not to show that God cannot know future contingencies, but to show that, just because God does know all the future, none of it can *be* contingent).

What, then, with all these lines of escape stopped up, are we left with? Nothing, Thomas thinks, but to accept the objector's conclusion; only with a careful elucidation of the exact sense in which it is true. Here I must do a little retranslating of his Latin. Above, I have used the phrase 'has come to God's knowledge' to render Thomas's *est scitum a Deo*, because this translation brings out the pastness of the *scitum*, on which the objection as stated so heavily depends. But in fact what is *scitum a Deo* is necessarily so whether this *scitum* expresses a *past* fact or an *eternal* one, and this point is explicitly admitted by Thomas when he restates this objection in the *Summa Theologica*, Part I, Question 14, Article 13, Objection 2. His answer to the argument, however, requires him to insist that what *is* in fact expressed, so far as we may suppose the antecedent to be true at all, is not proper pastness but eternity. And his answer consists in simply admitting that what is known to God *is* unalterable in the form in which God knows it; for God does not see the future contingent fact as future but as present. It is, he says, nearer the truth to say that if God knows a thing it *is*, than that if He knows it, it *will be*. At an earlier point, where he is neither stating nor

[1] Edwards, op. cit., Observation iii, Corollary i, discussion of Whitby. My attention was first drawn to this section in Edwards, and the resemblances between its opening argument and Thomas's Objection 7, by Mr. J. C. Thornton.

answering an objection but simply setting out his own view, he argues thus: The contingent, considered as future (*ut futurum est*) cannot be the object of any sort of knowledge which cannot fall into falsehood; so since the divine knowledge neither does nor can fall into falsehood, God could not possibly have any knowledge of future contingencies if He knows them *as* future. Divine *fore*knowledge of such events is, in fact, out; as He knows them, they are not still to come, but already there. This (which is what I had in mind when I said earlier that Thomas denies not only the logical proposition (10) but even in a way the theological proposition (8)) is a doctrine taken over from Boethius; its import is perhaps illuminated by the comment of an earlier follower of Boethius, namely Anselm, who observes that the unchanging 'presence' which on this view all things have to God, is in some ways less like our own present than our past. Looking back over what *has* happened, we can distinguish what was bound to happen as it did from what could have happened otherwise, though of course none of it *can now*, by the time we look back on it, have happened otherwise. It is in some such way as this that God distinguishes necessities and contingencies even though there is no contingency left in the latter in the form in which they reach His gaze.

For myself, I cannot wholly agree either with the objection or with Thomas's answer to it. I do agree with both that in some sense in which we *can* alter the future we *cannot* alter the past. But there is an objection to this that the future is precisely whatever it is that does come to pass after our alleged alteration has taken place, so what we alter *isn't* the future after all, and the real future can no more be altered than the past can. What I want to say to this—and as far as it goes I think it is Thomist doctrine too—is that nothing can be said to be truly 'going-to-happen' (*futurum*) until it is so 'present in its causes' as to be beyond stopping; until that happens, neither 'It will be the case that *p*' nor 'It will be the case that not *p*' is strictly speaking true. What Thomas says is that neither of them is true *determinate*; and what this appears to mean is that though they somehow share truth and falsehood between them, neither is as yet definitely attached to either proposition rather than the other.[1] I don't

[1] See, especially, the latter part of part i, *Lectio* 13, in his *Peri Hermeneias* commentary.

myself now think—though I once did—that this complication is necessary; it is enough to distinguish (as Thomas did not) between the form 'It will be that it is not the case that *p*' (which commits one to the futurition of not-*p*) and the form 'It is not the case that it will be that *p*' (which could also be true if it is simply as yet undetermined whether it is *p* or not-*p* that the future holds). Writing '*F*' for the simple 'It will be that', '*N*' for 'Not', and '*AXY*' for 'Either *X* or *Y*', I would say that we have at this stage

$$NAFpFNp$$

('Neither it-will-be-that *p* nor it-will-be-that not-*p*'). And this state of affairs we can alter, changing it to

$$AFpFNp$$

when it is in our power to decide one way or the other and we do so. But what is past cannot be thus altered, for it is *always* the case that either *p has been* the case or not-*p* has, i.e. we always have

$$APpPNp,$$

and there can be no question of changing from this to its opposite or vice versa. Moreover, with respect to any specific past time, say *n* time-units ago, we have

$$APnpPnNp,$$

but for some future times we have, on the contrary,

$$NAFnpFnNp.$$

Let us now put '*MFnp*' for the assertion that *p* is one of the things that *can* happen *n* time-units hence, and take this to mean that it *isn't* yet settled that *p* will *not* be the case at that time; i.e.

(vi) $MFnp = NFnNp.$

We can similarly define 'It can be that *p has* happened *n* time-units ago', *MPnp*, as *NPnNp*; but there is a very big difference between this case and the preceding. For 'It *isn't* the case that *p* was then *not* the case' is true only of those times of which it *is* the case that *p* then *was* the case,[1] i.e. we have

(vii) $MPnp = Pnp,$

[1] At least the theory of future contingencies provides no exceptions to this. For the possibility of other exceptions, see *Summa Theologica*, part i, Question 16, Article 7, Objection 4, and answer; and my own *Time and Modality*, ch. 4, and 'Identifiable Individuals', *Review of Metaphysics* (June 1960), pp. 692, 695–6.

whereas 'It isn't (yet) the case that *p* will then not be the case' can be true of 'thens' of which it isn't yet the case, either, that *p* will then *be* the case, i.e. we do *not* have as a law

$$MFnp = Fnp.$$

So I want to say that 'It can be that *X*' is logically equivalent to the simple *X* where *X* is a past-tense proposition, but not where it is a future-tense. Thomas and his objector would, I think, agree with this, but they say, further, that 'It *must* be that *X*' is equivalent to the simple *X* where *X* is in the past tense and not where it is future; and this difference I cannot myself obtain in any straightforward way.[1] What I have succeeded in formalizing is in fact not quite the Aristotelian-Thomist account of this whole situation, but a slight modification of it that you get in C. S. Peirce, who says that the past is the region of 'brute fact', while the future divides into the necessitated, for which alone we have either *Fnp* or *FnNp*, and the merely possible, for which we have neither.[2]

Still, with this position also the proposed distinction between past and future can be shown to break down if we equate *p* either with *PnFnp* or with *PnGFnp*, at least if we also admit (as all writers that I know of do) that

(viii) $p = FnPnp$,

i.e. a proposition *is* true if and only if it *will* be the case at any given time hence that it *was* the case that interval of time before. For on these assumptions we have

$$\begin{aligned} MFnp &= NFnNp && \text{(vi)} \\ &= FnPnNFnNp && \text{(viii)} \\ &= FnNPnNNFnNp && \text{(vii)} \\ &= FnNPnFnNp && (NNp = p) \\ &= FnNNp && \text{(iv)} \\ &= Fnp && (NNp = p) \end{aligned}$$

('$= FnNPnGFnNp$' may be inserted after the fourth line by (v) and removed by (iii)). Intuitively, the argument proceeds thus: suppose it is now possible that a certain thing, say *p*, should

[1] For the difficulty here, see my *Time and Modality*, p. 97.

[2] *Collected Papers* of C. S. Peirce, 5, 459.

come to pass *n* time-units hence. Then it *will* be true when that time comes (whatever actually happens then) that this thing *was* possible now. That is, it *will* be false then that the thing *was* at this present time booked to fail to come to pass. But if this will be false then, it will also be false then that it *is* failing to come to pass (for on the hypothesis that we are considering, if it were then failing to come to pass, it *would* now have been going to fail). But if it will then be *false* that it *isn't* coming to pass, it will be *true* that it *is* coming to pass. That is, from the mere possibility of a future event we can by these steps infer that it will actually occur. On this view also, then, the reality of future contingency is incompatible with our proposition (10), and by the same type of argument with proposition (8).

There is an interesting, and formally rather beautiful, relation between the 'tense logic' here advocated and the 'Occamist' tense logic mentioned earlier as an alternative to Thomas's. We can formalize the Occamist system by having one set of variables, say '*p*', '*q*', '*r*', etc., for statements generally, and a special further set, say '*a*', '*b*', '*c*', etc., restricted to statements with no trace of futurity in them. We might then have '$a = La$' as a law but not the more general '$p = Lp$' and not even '$Pnp = LPnp$', though we would have '$Pna = LPna$'. Certain functions of the '*A*-variables' would be substitutable for them in laws (would constitute '*A*-formulae'), others not. For example '*Pna*' as well as the plain '*a*' (and '*Na*') would count as an *A*-formula, and be substitutable for '*a*' in laws, but '*Fna*' would not, nor would '*PmFna*', though both of these would be substitutable for '*p*'. ('*Pna*' would be an *A*-formula because formed by prefixing '*Pn*' to an *A*-formula, but '*PmFna*' would not, because the formula to which the '*Pm*' is here prefixed isn't one.) '*LFna*' and '*LPmFna*' are of course well-formed, and propositions of this form could sometimes be true; and there is a case for counting as an '*A*-formula', i.e. as not having *proper* futurity, any formula at all that begins with '*L*', even ones like '*LFna*'. (Such assertions 'haven't proper futurity' because whether it is or is not necessary that *Fna* must depend solely on factors now in being, which either do or do not now leave open an alternative future, *n* time-units hence, to *a*.) And earlier I have in effect sketched a case for taking the same line with '*G*', and for having in this sort of system the law '$a = Ga$', but not '$p = Gp$'.

The laws of this system would include both '*NPnp* = *PnNp*' ('It is not the case that *p* was then so, if and only if it was then the case that not-*p*') and '*NFnp* = *FnNp*' ('It is not the case that *p* will then be, if and only if it will then be that not *p*'), and of course the substitutions of '*a*' for '*p*' in these. But whereas they will include '*NLPna* = *LPnNa*' (by the law '*a* = *La*', the fact that '*Pna*' and '*PnNa*' are *A*-formulae, and a preceding equivalence, these giving us the chain '*NLPna* = *NPna* = *PnNa* = *LPnNa*'), they will not include '*NLFna* = *LFnNa*' (the chain breaks because '*Fna*' and '*FnNa*' are not substitutable for '*a*' in '*a* = *La*').

Suppose now we remove the Occamist's functor '*F*' from the system and replace it by another '*F*' equivalent to the Occamist's '*LF*'. Assuming that there is no way of forming 'non-*A*' propositions out of the *A* ones except by the use of the Occamist '*F*', in this new system none but *A*-propositions will be formulable, so no variables need be used but *A*-variables; or if you like, the *P*-variables can be treated as *A*-variables. Because we had '*a* = *La*' in the old system, we will have '*p* = *Lp*' in the new one; in fact there will be no use in it for the operator '*L*'. And we will have '*NPnp* = *PnNp*' in the new system as in the old; but we will *not* have '*NFnp* = *FnNp*' for the new *F* is the old '*LF*', and we didn't have '*NLFna* = *LFnNa*'. We will, though, now have '*p* = *Gp*', since we had '*a* = *Ga*' in the old.

This 'new' system is in fact precisely the 'Peircean' or near-Aristotelian system advocated above. So it could be said, and indeed it has been said (e.g. by Professor Shorter), that the system advocated is merely the Occamist one robbed of its means of expressing contingent truths. I would reply that in an important sense of 'truths' there are no contingent truths; once a thing reaches the status of a 'truth' there can be no going back on it; though there are 'contingencies', i.e. matters of which it is not yet true either that they will be the case or that they will fail to be the case. This is of course a terminological difference rather than one of substance, but being a difference as to what we shall count as a 'truth', it affects what we mean by 'God knows all truths', and so could (and in my view should) affect what truth-value we attach to this statement.

There is, we may observe at this point, an even more direct way of getting '*p* = *Lp*' than the method of Thomas's Objection 7.

For the schoolmen commonly contrasted the contingency of the future with the necessity not only of the past but also of the present—not only what *has been* the case cannot now not have been the case, but what *is* the case cannot now not *be* the case. But it is plausible to say that the functor 'It is the case that—' makes no difference to the truth or falsehood of *anything* to which it is prefixed, so that *all* propositions are equivalent to ones which are of the present tense in their principal clause. Thomas seems to admit this when, commenting on *De Interpretatione* 16 b 17–19, he equates *est praeteritum* and *fuit praesens*, and *est futurum* and *erit praesens*—putting '*Sp*' for 'It is the case that *p*', we have '$Sp = p$' and in particular '$SPp = Pp = PSp$' and '$SFp = Fp = FSp$'. And this, given '$Sp = LSp$', gives '$Fp = SFp = LSFp = LFp$', thus breaking down the above difference between future and present. The Occamist answer to this is presumably to replace '*p*' by '*a*' in the law '$Sp = LSp$' (there is then no need for him to alter '$p = Sp$'); the Peircean admits the conclusion but gets the reality of future and unreality of present and past contingency in another way.

As to Thomas's own answer to his real or imaginary objector, I can only say this: I simply cannot see how the presentness, pastness or futurity of any state of affairs can be in any way relative to the *persons to whom* this state of affairs is known.[1] What makes this quite impossible to stomach is precisely the truth that both Thomas and his objector insist on, namely that the future has an openness to alternatives which the past has not; such openness is just not the sort of thing that can be present for one observer and absent for another—either it exists or it doesn't, and there's an end to it; and so either a thing has already occurred or it hasn't, and there's an end to *that*. But the presentness, pastness or futurity of states of affairs does of course vary with *time*, i.e. it is itself a tensed matter—what *was* future or present, *is* now and *will be* past, and so on.[2] So I don't understand what is meant by saying that contingent future occurrences are neither contingent nor future *as* God sees them, though I do understand what would be meant if it were said that they are neither contingent nor future *when* God

[1] Cf. Scotus, as given in Ockham, op. cit., p. 53 and n.

[2] Cf. McTaggart, *The Nature of Existence*, ch. xxxiii, Sects. 305, 330; and my own 'Time after Time', *Mind* (April 1958), pp. 244–6.

sees them. How, in fact, could God *know* a state of affairs to be present and beyond alteration, until it *is* present and beyond alteration (for if He sees it as present when it is not, surely He is in error)? But to know that something is so when it is so, is surely not foreknowledge. So when I try to set out to myself what Boethius and Thomas—and later on, Peter de Rivo (and even Peirce)[1]—are saying here, I find that either I cannot understand what I am saying, or I slip into something which I certainly *can* understand, but which is surely too trivial altogether to express the intention of these writers. Still, with this trivial thing, so far as it goes, I do agree; I agree, that is, that God, or let us say any omniscient being, knows what is happening; and of course I agree also with the negative admission of Thomas and of Peter de Rivo that God *doesn't* know future contingencies literally *when* they are still future and contingent, and that it is impossible that He or anyone else should know them in this way. But (and this is what Thomas himself says[2]) this is only because there is not then any truth of the form 'It will be the case that *p*' (or 'It will be the case that not *p*'), with respect to this future contingency *p*, for Him to know; and *nihil potest sciri nisi verum.*

[1] C. S. Peirce, op. cit., 4, 67.
[2] *De Veritate*, Question 2, Article 13, Objection 1 and answer.

IV

CONTEMPLATION AND ACTION

IT would be easy to cavil at this title 'Contemplation and Action' if it is meant to suggest some kind of contrast, as surely it is. For whatever contemplation is, it is among other things a kind of action, for it is something which men do. One might meet this easily enough, of course, by revising the title to 'Contemplation and Other Actions'. But it is not as simple as this—there are contexts in which 'Contemplation and Action' *is* the right antithesis. If we are going to talk about the merits of different kinds of life, then contemplation is one among the various things we may habitually put our time to, and we ought to talk about 'contemplation and other actions', or, better, about 'contemplation and other activities'. But if we have before us some sort of image of a man at some cross-roads, first viewing the situation, sizing it up, and then making up his mind what to do about it, a contrast between 'contemplation' and 'action' is entirely in order; we can almost *see* the dividing-line between the two. And I propose to concentrate on this, the topic which the title properly suggests.

In this context one makes a sharp distinction between contemplation and action because one makes a distinction between what is within one's power and what is not. The shape of the future depends on what *we* do with what is *given* to us, or to be a little less anthropomorphic, on what *we* do with what is *there;* and by 'what is *there*' we mean basically what has already happened, together with what is already being carried forward, as it were, by a certain momentum from the past, too great for us to stop or divert.

We make this distinction all the time, do we not? Even in public life, we set up 'fact-finding commissions', whose job is as it were purely contemplative—setting the situation clearly before whatever body it is that does the quite different thing, acting. And a good deal of practical wisdom consists, or at all

events seems to consist, in ascertaining just where we can do no more than contemplate, and where our freedom of action begins. Moreover, there seems to be some sort of *inconsistency* between being an object of contemplation and being an object of action; if there is something anywhere to be seen and contemplated, then that is beyond being determined by our action, while if there is something which we can bring about or prevent by our action, then that thing is to that extent opaque to the contemplative eye. I can look in the larder now and see what is there, and over what I shall see I have no control; I can perhaps also look in an almanac and 'see' where the moon will be at 9 o'clock tomorrow night; but I cannot in the same sense 'see' what I shall eat for supper tonight—I may eat what is in the larder, or eat out, or not eat at all; but it is not a question of *finding out* what I shall do, but of deciding. In order to decide rationally, there is no doubt much that I must find out, for example, what restaurants will be open and whose they are, but It would be in a hopeless position if one of the things I had to find out was *how I shall decide.*

At this point, then, contemplation and action seem as sharply distinguished as they could be. But reflection suggests that it is not as simple as all that. This 'momentum' from the past at least sometimes carries human actions along with it as well as other things, and this lack or possible lack of freedom in our choices is not just a speculative matter. In sizing up the situations before us we include an estimate of how other people will decide, and they in turn treat our decisions thus. And at some points we do seem even explicitly to treat our own decisions thus; e.g. when we say 'I prefer political candidate X to either Y or Z, but he hasn't a chance of gaining enough votes, so if I vote for him I'll only weaken the chances of Y, who's the better of the other two, so I'll vote for Y'; or even this: 'I prefer X to either Y or Z, and so do most people, but they're afraid to vote for him because they think they'll do no more by it than splitting the votes for Y; so if I vote for him, etc.' How nice (and contemplative) the 'they' is here!—'I would vote for him if I thought it would do any good; but unfortunately *they* won't vote for him because *they* don't think it would do any good; so it won't do any good, and I can't vote for him.' Still, this is a rational consideration, isn't it, when one is deciding how to vote?

Nevertheless, the direct 'I'll find out how I'm going to decide, and then decide that way' has an odd sound; and no wonder, for the grammar of 'decide' is such that it would be just as sensible to say 'I'll find out how I'm going to decide, and then decide the other way'. The first, indeed, looks as if it *could* be true, while the second could not, if the words 'and then' were taken quite literally as expressing temporal succession and nothing more. But if I really do *find out what I am going to do*, can I really be said, after having found this out, to *decide* to do it? and consequently can I be said to have *found out what I am going to decide?* The answer to these questions seems to me quite clearly No. Contemplation and action at this point really are incompatible.

But might not *somebody else* find out how I am going to decide? This is not so easy to answer. The person might tell me his findings, but then I might not believe him, or might at any rate not know that he is right, and so not myself 'find out' what I shall do; and so I might still really 'decide' after this other person had found out. He might, too, tell me falsely what he had found with the very idea of making me do the other thing to prove him wrong, this other thing being what he foresaw all along that I should do. I could not rationally think of myself as 'deciding as he had *found out* that I would not decide', but only as 'deciding as he had *thought* that I would not decide'; but he might really have found out, nevertheless (i.e. found out that I would decide, not as he said, but as I did decide).

If one takes this view, however, one is committed to the view that action, in the sense of decision, presupposes a certain amount of ignorance—ignorance of what might in principle be known, and perhaps even *is* known to someone other than the agent. A man may, of course, on this or any view, 'know how he will act' *after* he has decided; but it is not 'decision' if he knows it before (there may still be *effort* in this case, but that is another thing). From this it seems to follow that the power to make decisions is something not entirely desirable. For if, say, someone has found out what I shall do and has told me, it would be better for me to know that what this man says is true and so cease to be able to 'decide' about it, than for me to remain in my ignorance and then decide as the man said I would and knew I would. 'Poor chap!' he might well say, 'I

knew what he would do, and told him, but he still had to go through all that business of deciding.'

But I fancy that part of our feeling that this would be a proper reaction on the wiser man's part, arises from a suspicion, it may be even a conviction, that in such a case the 'business of deciding' would have something spurious about it—in view of the other man's knowledge, it all seems shadow-play and not real deciding; I would think this about it myself if the man convinced me later that he really did know what I was going to do. Suppose, again, that the man says I will do X and just because he says so I do Y; and he then says 'I knew you would do that; in fact I knew that my saying you would do the other thing would make you do that, and that was why I said it'. My first reaction would no doubt be to say, 'Look here, if I'd known that I wouldn't have done it.' But this would rather disprove my freedom than prove it—the other man knew all that too. I would be left feeling, I think, a rather push-button creature; or at least I would feel that my behaviour on the occasion in question had been push-button behaviour and not the 'decision' I had taken it to be at the time.

So I want to say, now, that 'how I shall decide' is something which not only cannot be known beforehand (cannot be 'contemplated') by myself, but cannot be known beforehand by anyone else either, because if it is a genuine decision there is *nothing to be known* beforehand about which way it will go. This much, it seems to me, is involved in the way we ordinarily use the verb 'decide'. It is therefore a consequence of this ordinary use of the verb 'decide' that we cannot consistently talk of 'contemplating' a person's future decisions (either one's own or someone else's). Or rather, we cannot contemplate *specific* future decisions, e.g. 'the decision that Jones is going to make, to have tea in town tonight'; though one can contemplate such decisions described *generically*, e.g. I can contemplate 'the decision that Jones is going to make about whether to have tea in town tonight or not', and reflect about it that if he does not make it before 4.30 p.m. he will be too late for the last bus in by tea-time. I cannot contemplate the decision Jones is going to make to have tea in town tonight, because there is just no such thing as the decision Jones is going to make to have tea in town tonight; nor any such thing as the decision he is going to make

not to have it. After it is all over, the decision Jones is going to make about whether to have etc. *will have become* either a decision-to-have or a decision-not-to, and will then be contemplatable as such. Or alternatively, Jones *is* going to have tea in town tonight, or he is not going to, and I can contemplate that (whichever it is), but *in this case there is just no (future) 'decision' about it.*

All this is I think involved in the ordinary use of 'decide'. It is also implied in much that we ordinarily say that 'decisions' in this sense do from time to time occur. Whether this is really so I do not know; the fact that it is commonly assumed, and the assumption reflected in common speech, is no argument one way or the other.

To sum up: It may be said that contemplation without action is impotent and action without contemplation is blind; but such impotence and blindness are inescapable if contemplation is to be contemplation and action action. Only it is possible that there is in fact no action but only happening, some of which feels like action but isn't.

Extract from discussion of this paper (involving J. L. Mackie, K. Baier, and J. A. Passmore):

MACKIE: I agree that it is absurd to say 'I know that I shall do *X*, so now I decide to do it'; it is like 'I know the solution to this problem, so now I shall proceed to solve it'. But there's nothing wrong with 'I know the solution to this problem, and now *he* will proceed to solve it'; nor is there with 'I know that he will do *X*, and now he will decide to do it'.

PRIOR: Solving a theoretical problem and deciding what to do are alike in that each is the closing of a gap, and cannot take place if the gap is already filled. With the problem-solving it's just a gap in a man's knowledge; in the other case it's a *gap in the facts.* When it isn't yet the case that I am going to do *X*, and isn't yet the case that I'm going to refrain from doing *X*, my decision is needed to make one of these things the case. But if it isn't yet the case that I shall do *X* I can't know that I shall do *X, and neither can anyone else;* similarly with the other alternative.

BAIER: But 'being the case' isn't the sort of thing that it makes sense to attach a time-qualification to.

PRIOR: Why not?—in common speech we say that whereas yesterday Jones was a hungry man today this is no longer the case.

PASSMORE: In common speech we also say 'I know how he is going to decide'.

PRIOR: Common speech has a strong and a weak sense of 'decide'.

MACKIE: Suppose you take yourself to be making, over a long period, a series of 'decisions', and then I suddenly come forth with a certain Mr. Smith, who all the time has been predicting what you will do, and has been right every time. Would this give you any inclination to withdraw the claim to have been making decisions?

PRIOR: Yes.

MACKIE: So that it *might* be that you never make decisions at all, in your sense of 'decision'?

PRIOR: Yes.

V

THE CONSEQUENCES OF ACTIONS

In Moore's *Principia Ethica* it is repeatedly affirmed that our duty is that action which, of all the alternatives open to us, will have the best total consequences. Moore himself emphasizes in this work the practical impossibility of finding out with any certainty what our duty is, given this definition of 'duty'. I do not want to go into that; what I want to do now is to argue that there is not merely a practical impossibility in finding out what our duty in this sense is, but something more like a logical impossibility in there *being* such a thing as a duty in this sense.

My argument is dilemmatic. Either determinism is true or it is not. If determinism *is* true then there are not really (though there may seem to be) a number of alternative actions which we could perform on a given occasion; the one action that we can perform is the one that we do perform. Hence whatever we in fact do is the best possible action (the one with the best possible total consequences) because it is the *only* possible action; so that whatever we in fact do is our duty, in Moore's sense of 'duty'. Moore himself saw this horn of the dilemma (and indeed it is a commonplace that determinism presents problems of this sort); but it has another horn which so far as I know he did *not* see. Suppose that determinism is *not* true. Then there may indeed be a number of alternative actions which we could perform on a given occasion, but none of these actions can be said to have any 'total consequences', or to bring about a definite state of the world which is better than any other that might be brought about by other choices. For we may presume that other agents are free beside the one who is on the given occasion deciding what he ought to do, and the total future state of the world depends on how these others choose as well as on how the given person chooses; and even if there were not other people to spoil one's calculations there would still be

oneself, with one's own future choices, or some of them, undetermined like this present one (unless a man decides that it is too risky for him to have any further free-will, and on this very ground finds it to be his duty to do away with himself). And while I speak here of one's calculations being spoilt, the trouble of course goes deeper than that—it's not merely that one cannot calculate the totality of what will happen if one decides in a certain way; the point is rather that there *is* no such totality.

The conclusion seems clear. If determinism is true, then whatever we do is our duty in Moore's sense of 'duty', and if determinism is not true then nothing at all is our duty in this sense. Hence, if we use the term 'duty' in this way, either whatever we do is our duty or nothing at all is our duty, and either way there cannot be any duty that we have failed to perform. We are driven, in a word, to a version of what Professor G. E. Hughes calls the Principle of Continuous Moral Rectitude. This principle is, indeed, only deduced with a certain proviso. It is possible that someone should fail to perform a duty in the sense of *Principia Ethica* if this agent (i) is free, and (ii) is the only free agent there is, or at least the only free agent there will be from the time he makes his decision, and (iii) himself acts freely only once in the course of his existence. This complex supposition is of course one which no reasonable person would make, but it is of some interest that this supposition alone prevents the deduction of the Principle of Continuous Moral Rectitude from Moore's definition of 'duty'. For such moral solipsism would amount to treating oneself as a kind of God (with the problem about whether to tie one's own hands or not with which God is traditionally beset); and one of Butler's criticisms of Utilitarianism was precisely that it involved a rash assumption of a divine prerogative.

It is not only over the position of *Principia Ethica*, however, that this sword hangs. Our deduction would obviously be equally possible if we equated 'doing our duty' with securing the best possible total consequences, not by definition as in *Principia Ethica*, but synthetically as in Moore's later *Ethics*. And our paradox is only slightly mitigated if we adopt a position which is not as a whole Utilitarian at all, but which admits the securing of the best possible consequences as one among a number of prima facie duties. For if we ascribe a certain finite

weight or degree of stringency to a prima facie duty which either is something that is always automatically performed or is just nothing at all, a senseless or indeterminate element is introduced into the whole complex of prima facie duties of which our duty *sans phrase* is supposed to be the resultant.

It really is necessary, I think, for the Utilitarian, and even for the man who makes the maximization of goodness a prima facie duty, to climb down a little here, but how little can we make it? Taking the non-determinist horn first, perhaps we can say that if determinism is not true, it suffices to speak of a duty to do what will *probably* have the best total consequences of all the actions open to us. We can only take this line, however, if we are prepared to talk about objective probabilities; that is, if we are prepared to argue that '*p* is probable' need not merely mean 'We don't know that *p* will be true, but what evidence we have is more in favour of it than against it', but may mean something more like '*p* is not yet either going to be the case or not going to be the case, but is more like going to be the case than not'.

What about the determinist horn? Even apart from my use of it to construct this paradox, many determinists are irritated by the suggestion that their position is incompatible with the view that there are alternative possible courses open to us in given situations. They will insist that there are numerous quite ordinary senses of 'possible' in which determinism is not at all incompatible with this. Moore in his *Ethics* gives the example of two ships one of which *could* be going at 20 knots and the other of which could not, though neither of them is in fact doing so, and neither of them would be credited with free will. It is clear that we arrive at this sort of possibility by deliberately leaving certain factors out—as far as the construction of its engines, say, is concerned, this ship which is not doing 20 knots *could* be doing 20 knots—that is, in circumstances which are quite ordinary, though they do not in fact obtain at this moment, ships with that sort of engine *do* go at 20 knots. And so far as I can see, all the senses of 'could' which the determinist admits are of this general type. But while this may help the determinist in some of his troubles, it surely does not help him here. For how can we seriously estimate the *total* consequences of an action when we have deliberately left out of account certain

of its surrounding circumstances? At least some of the features of the situation which results after my action are bound to be due to the very factors which in the end made me act as I did, and which I therefore had to ignore when making my decision, and treating it *as* a decision.

What is substantially the same point might also be expressed as follows: even on a determinist view, the succession of situations which follow one another after a given action has been performed is never the result of that action alone, but of that action together with an infinity of other concurrent happenings. It is quite impossible to draw a sharp line and say that everything on one side of it is a consequence of the action, and everything on the other side a consequence of other things; and therefore impossible to arrive at a set of total consequences of the action. What makes it momentarily appear to be possible is that we do sometimes say of an action X, 'If X had not been done, Z would not have happened'. But this is not such a simple hypothetical as it seems—in practice what we always mean is 'If X had not been done, but Y had been as it actually was, Z would not have happened'. This would still enable us to describe Z in ordinary parlance as a 'consequence' of X; but when it comes to specifying X's *total* consequences the antecedent of our hypothetical must be 'If X had not been done, but *everything* else had been as it actually was . . .', and on a determinist view this is an impossible supposition. The notion of the *total* consequences of an action seems thus to suffer from an incurable incoherence which renders it useless for ethical theory or for any other sort of theory.

Now this really ought, so far as I can see, to be the end of the matter. But I am aware that abstract logic is for some reason a little suspect these days; so out of deference to this attitude, let us shut up our Moore for a while, and take down Mother Goose.

For want of a nail
  The shoe was lost;
For want of a shoe
  The horse was lost;
For want of a horse
  The rider was lost;
For want of a rider
  The battle was lost;

For want of a battle
The kingdom was lost;
And all for the want
Of a horse-shoe nail.

In these lines we are asked, in effect, to assent to the validity of the following sort of reasoning: The fate of the kingdom depended on that battle (it was what is called a 'decisive' battle); one more good cavalryman in the field would have saved the day; we'd have had Bayard Bloggs there if his horse hadn't been crippled through the loss of a shoe; the shoe would have held out if it had had one more nail in it; so that nail lost us the kingdom, and plague take all careless blacksmiths. Well, no doubt the smith's negligence was most reprehensible—we all know the importance in warfare of what is called a 'high standard of maintenance'—but it is surely a little hard to place on his shoulders *the* responsibility for the loss of the kingdom. If Bayard Bloggs's absence could make that much difference, the battle was clearly a close go, and an extra cannon might have turned the scale as well; perhaps we would have had it too, if the horse that was pulling it hadn't given in through exhaustion; that horse would have been in better fettle if it had been better fed; it would have been better fed if its master hadn't had that bout of 'flu, which he would not have had if he hadn't been courting Mary Jane on Ilkley Moor without a hat; he'd have had the hat if his brother Bert hadn't borrowed it; so the whole thing was Bert's fault really, rather than the blacksmith's.

The loss of the kingdom *was*, all the same, a 'consequence' of the blacksmith's negligence, even if it was what we would call a 'remote' consequence; and it would certainly be included among its 'total' consequences by those who use this phrase. But so would much else; for example, if the horse, too late for the battle because of its injury, became the ancestor of a breed of racehorses which brought more renown and profit to the kingdom than it had ever had before it was conquered—this must stand among the 'consequences' of the blacksmith's negligence also, though it would hardly be considered a ground for conferring a knighthood on a man. Sooner or later, in fact, the consequences of the blacksmith's negligence—and also those of Bert's borrowing the hat—will include the entire international situation.

So far, I should say, our reflection on the nursery story has uncovered nothing that tells *against* Moore's account of what duty is. In considering concrete cases one always becomes acutely aware of how little anyone *knows* about the consequences of one's actions; but this is only part of the general problem of 'duty and ignorance of fact', which has nothing specially to do with Utilitarianism, and was allowed for by Moore anyway. Our responsibility is to do our duty as far as we can see it, and unless something is a duty whether we can find it out or not, we cannot even 'do our duty as far as we can see it', for there is nothing for us even to start looking for. If the blacksmith's foresight had extended to the breed of racehorses, perhaps he *would* have deserved a knighthood.

But what about Bert's borrowed hat? A disciple of Moore could say, I think, that that was one of the things the blacksmith needed to know about in order to know his duty fully, but it did not make that duty any the less his, the blacksmith's, duty. There is a passage in Kierkegaard's *Purify Your Hearts* which is perhaps relevant here. Kierkegaard says that too many people listen to sermons as if they were theatrical performances —as if the preacher were a sort of actor, to be subjected after the service to discerning dramatic criticism. In fact, he says, it is we, the listeners, who are the actors, and the preacher is rather a kind of prompter; the responsibility of each of us is to look to *our own* acting, rather than judge anyone else's. So the blacksmith must not judge the hat-borrower, but simply see to his own duty in the circumstances that the hat-borrower has placed him in; and the hat-borrower similarly *vis-à-vis* the blacksmith.

But wouldn't a man go mad if he really tried to take the whole responsibility of everything upon himself in this way? And suppose I tell you, now, that the blacksmith's negligence in the matter of the nail was a result of sheer intolerable fatigue, brought about by the absence of the young man who usually assisted him. And where was this young man? Why, away at the wars, of course—and in that very battle of which we have been speaking. (To anyone complaining of the deterioration of his standards of workmanship, the blacksmith would have said, 'Don't you know there's a war on?') The blacksmith *couldn't* have gone on and on and on shoeing soldiers' horses as he was

expected to without his assistant by his side; but if this same assistant had been out of the battle, perhaps even Bayard Bloggs on his charger could not have saved the day. This surely puts the blacksmith's duty, in Moore's sense of 'duty', in a very queer light indeed. The urgency of equipping Bayard Bloggs's horse satisfactorily—its being a matter which *could* decide the battle—depends in part on the very thing that makes it impossible that the blacksmith *should* perform this duty. It is difficult to resist the temptation to cry out at this point, 'But surely we cannot suppose the smith to know about this twist to the thing', as if that were relevant to the present discussion. Of course it is not relevant—whether we can suppose the smith to know about it or not, the whereabouts of his assistant is one of the factors that makes the smith's actual or objective duty what it is (on Moore's view of the matter) and at the same time determines his capacity to perform it.

Of course when we bring up this sort of thing we are ceasing to take the blacksmith seriously as a person who *has* duties. The responsibility has passed now to somebody like the Minister in charge of manpower; and I don't know whether even this is taking it far enough back. For the Minister also is in an awkward position. We have some inclination to say that the military disaster was a consequence of his drafting the blacksmith's assistant, so that the blacksmith was too tired to notice the missing nail, Bloggs could not arrive on time, and so on. But if he had *not* drafted the blacksmith's assistant, we would have been inclined to call the defeat a consequence of *this* fact. There is something that seems to bear on this sort of dilemma in the *Prior Analytics*. Aristotle argues that q cannot ever be necessitated both by p and not-p, because

(1) p necessitates q, and not-p necessitates q

entails

(2) not-q necessitates not-p necessitates q,

by contraposition of the first conjunct; and this in turn entails

(3) not-q necessitates q

by hypothetical syllogism, but (3) is a fantastic proposition. This is nowadays generally written off as an error, and with some senses of 'necessitates' it *is* an error, but how does it work out in the present case? We suppose that defeat is a consequence

of the drafting of the blacksmith's assistant, but would equally have been a consequence of not drafting him. Then we can say both that to avoid defeat the Minister should not have drafted the blacksmith's assistant, and that if he had not drafted him defeat would have followed, so that defeat would have followed from his avoiding defeat. This conclusion certainly sounds odd; yet something like it seems to express exactly how the Minister *is* placed. Whichever measure he might take, and *rationally* take, to avoid defeat, will result in defeat—this *is* the fix he is in, is it not? And of course the upshot of the whole matter, both formally and concretely, is that defeat is inevitable.

It is inevitable, anyway, if the only pieces that the Minister is free to move are Bloggs and the blacksmith's assistant, and he certainly could be in that position at some stage. But if we are to regard the defeat as in any sense capable of being weighed against other alternatives, i.e. against the consequences of other alternative previous courses, we must say that either the Minister himself at an earlier stage, or somebody else, might have disposed the country's forces in a way that would not have left the position so desperate. If only, earlier on, the Minister had taken a *little* less whisky at that diplomatic party—. But a man must relax sometimes, mustn't he?—I mean, even to do his work effectively he must. What has been done with the blacksmith can be done with the Minister too, and it is clear that we should soon be back where we were before. The notion of 'total consequences', in short, will not work any better in the concrete than it will in the abstract; and the concrete case perhaps makes it clearer than ever that it is even more unworkable on the determinist hypothesis than on the indeterminist one. The indeterminist can approximate to it with the notion of '(objectively) probable total consequences', but the determinist cannot sort out one action from another sufficiently sharply to give us even that.

VI

LIMITED INDETERMINISM

My main business here is the examination of a certain argument of Jonathan Edwards. There seems to be something of an 'Edwards revival' among at least a section of American philosophers, and this is a piece of New World patriotism with which I have considerable sympathy. Edwards was, as it happens, one of the first philosophers I ever heard of, and I still think he bears reading. For there is in his work, beside his crude rationalistic psychology, and beside the half-submerged predestinarian fervour that is so strangely echoed and transmuted in Herman Melville's *Mardi* and *Moby Dick*, a certain metaphysical *logic* with which we may still grapple profitably. In particular, his treatise on the will has three closely reasoned excursuses which raise wide philosophical issues in a way that is still intriguing: the discussion of knowledge and contingency in Part II, Section XII; that of the identity of the nonexistent in Part IV, Section VIII; and the discussion in Part II, Section III, on which I wish to concentrate now.

The general question to which Edwards here addresses himself is 'whether any event whatsoever, and *volition* in particular, can come to pass *without a cause* of its existence', and among other arguments for a negative answer he has a *reductio ad absurdum*, arguing that if an act of will can occur without a cause, then anything at all, no matter how fantastic, can occur without a cause. There is, he says in effect, an inner contradiction in the notion that uncaused events are bound always to be acts of will. We must note, however, in following his argument through, that his language is not quite that which I have just used, and in particular he does not speak primarily of what 'occurs' but rather of what 'begins to be'. He says,

> What is self-existent must be from eternity . . .: but as to all things that *begin to be*, they are not self-existent, and therefore must have some foundation of their existence without themselves. That what-

soever begins to be, which before was not, must have a Cause why it then begins to exist, seems to be the first dictate of the common and natural sense which God hath implanted in the minds of all mankind.

Then he immediately subsumes what we might be inclined to regard as a different and less radical sort of change under this one as a special case.

This dictate of common sense equally respects substances and modes, or things and the manner and circumstances of things. Thus, if we see a body which has hitherto been at rest, start out of a state of rest, and begin to move, we do as naturally and necessarily suppose there is some Cause or reason of this new mode of existence, as of the existence of a body itself which had hitherto not existed. And so if a body, which had hitherto moved in a certain direction, should suddenly change the direction of its motion; or if it should put off its old figure, and take a new one; or change its colour: the beginning of these new modes is a new Event, and the mind of mankind necessarily supposes that there is some Cause or reason of them.

Then, after some of the usual remarks about the dependence of all but the most immediate forms of knowledge upon the acceptance of this principle, he argues that if there is a single uncaused event or thing there is no reason why there should not be an indefinite number. And indeed, he says, the advocates of free-will do hold that millions of uncaused events, namely volitions, are occurring all the time. But this argument can be developed qualitatively as well as quantitatively. There is not the least reason why uncaused events should be all of one *kind*, if there are uncaused events at all.

If it were so, that things only of one kind, *viz.* acts of the will, seemed to come to pass of themselves; . . . this very thing would demonstrate that there was some Cause of them, which made such a difference between this Event and others, and that they did not happen contingently. For contingence is blind, and does not pick and chuse for a particular sort of Events. Nothing has no choice. This No-Cause, which causes no existence, cannot cause the existence which comes to pass, to be of one particular sort only, distinguished from all others. Thus, that only one sort of matter drops out of the heavens, even water, . . . shows that . . . something besides mere contingence has a hand in the matter. If we should suppose Non-entity to be about to bring forth; and things were coming into existence, without any Cause or Antecedent, on which the existence,

or kind, or manner of existence depends; or which could at all determine whether the things should be stones, or stars, or beasts, or angels, or human bodies, or souls, or only some new motion or figure in natural bodies, or some new sensations in animals, or new ideas in the human understanding, or new volitions in the will; or any thing else of all the infinite number of possibles; then certainly it would not be expected, although many millions of millions of things are coming into existence in this manner, all over the face of the earth, that they should all be only of one particular kind, and that it should be thus in all ages.

And it just won't do—this is the nerve of his argument—to say that it is the peculiar and special *nature* of volitions to start into being without a cause, for *nothing has any nature until it is there, so that whatever a thing's nature may explain or permit, it cannot explain or permit the thing's starting to be.*

If any should imagine, that there is something in the sort of Event that renders it possible for it to come into existence without a Cause, and should say, that the free acts of the will are existences of an exceeding different nature from other things; by reason of which they may come into existence without any previous ground or reason of it, tho' other things cannot; . . . I would observe, that the particular nature of existence, be it never so diverse from others, can lay no foundation for that thing's coming into existence without a Cause; because to suppose this, would be to suppose the particular nature of existence to be a thing prior to the existence; and so a thing which makes way for existence, with such a circumstance, namely, without a cause or reason of existence. But that which in any respect makes way for a thing's coming into being, or for any manner or circumstance of its first existence, must be prior to the existence. The distinguished nature of the effect, which is something belonging to the effect, cannot have influence backward, to act before it is. The peculiar nature of that thing called volition, can do nothing, can have no influence, while it is not. And afterwards it is too late for its influence: for then the thing has made sure of existence already, without its help.

Hence Edwards concludes that

It is . . . as repugnant to reason, to suppose that an act of the will should come into existence without a cause, as to suppose the human soul, or an angel, or the globe of the earth, or the whole universe, should come into existence without a cause. And if once we allow, that such a sort of effect as a Volition may come to pass without a

Cause, how do we know but that many other sorts of effects may do so too?

It is clear that this argument, if it is any good at all, will apply not only to those who say that the nature of acts of will is such that they (and they only) can occur causelessly, but also to anyone who says that the nature of *any* sort of event, say an electron's 'jumping ' from one of a limited set of orbits to another, is such that events of this sort, and of this sort only, can occur causelessly. But *is* the argument any good?

Let us begin at, or at least near, the beginning, with Edwards's subsumption of all happenings, or anyhow of all changes, under the idea of the 'beginning to be' either of concrete objects or of abstract ones. It is easy to dismiss this as a mere quirk of language—a bit of harmless philosophical pedantry. And no doubt it is easy to attach a harmless sense to the statement that my headache, say, began to exist an hour ago—what does this mean but that an hour ago my head started to ache? And again, no modern writer on causation would deny that it is, as Edwards says, common sense to look for the cause of an event of this sort (one's head starting to ache), and not just to look for causes when we are confronted (if we ever are) with the absolute beginning-to-be of a 'thing'. Indeed, what seems odd to modern readers is not that Edwards should thus extend the principle of causation to motions, accelerations, colour-changes and the like, but that he should feel that this required special justification—that he should regard it as an'extension' at all—that he should take something else, the beginning-to-exist of a 'thing', as his paradigm of what requires a cause, and so feel obliged to interpret, for example, the imparting of an acceleration as giving 'existence' to a new 'mode of motion'.

But this inversion, I would contend, is not just a bit of 'quaintness' or backwoodsmanship—it is of the very first importance, and his argument depends on it. And so, at the risk of myself falling into philosophical pedantry, I shall proceed to knock it down. I have admitted that we can, if we like, describe a head's starting to ache as a headache's starting to exist; but what must be insisted upon, if we are to answer Edwards, is that this change of key is *not* metaphysically illuminating but metaphysically obfuscating. This 'existence' and this 'starting to exist' of things like headaches is a purely Pickwickian and

eliminable existence and starting-to-exist, and we explain what is meant by a headache's starting to exist by saying that it just means a head's starting to ache, not vice versa. Behind this view is, of course, the assumption that the world consists not of events, such as headaches, but of things, such as heads, which act and interact and change. And a remoter part of this underlying assumption is that how things behave—that is, what events occur—is determined partly by their natures or dispositions, and partly by what happens to them.

Given this metaphysical apparatus, it seems perfectly possible to say that some things, but not all things, have alternative possibilities of reaction to one and the same stimulation. It is 'open', we might say, to a disturbed electron to jump to orbit *A*, and equally open to it to jump to orbit *B*, but perhaps not open to it to jump to orbit *C*. In other words, its dispositions may be such that with certain provocations it will 'jump to orbit *A* or to orbit *B*', without having any determinate disposition to jump to orbit *A*, or any to jump to orbit *B*. Its jumping to orbit *A* rather than to orbit *B*, which we can call if we like the coming-to-be of a jump to orbit *A* rather than of one to orbit *B*, will then be a circumstance without a cause. And it may be that the only circumstances without causes are the ways that electrons jump from orbit to orbit. But the explanation of this fact will lie, not in the nature of those non-existent or not-yet-existent jumps, but in the nature of the *existing* electrons (and of other existing things). A similar explanation holds, if the only uncaused circumstances are the ways people choose; or if the only ones are of this sort *or* the preceding. We cannot and do not need to say that it is the nature of 'volitions', or of certain volitions, which makes their coming-to-be possible but not necessary, and the nature of other non-existent or not-yet-existent occurrences which makes their coming-to-be either necessary or impossible. It is rather that there are certain already-existing objects which have certain capacities, and some which lack them, and none which have certain other capacities. Persons, say, have the power, without the necessity, of doing *X* in certain circumstances; for oysters, on the other hand, doing *X* may be necessary or impossible; and *Y*, say turning into a dragon, may be something which no existing object has the power to do.

Don't be misled by this last way of talking either. A 'capacity' is not a *relation* between an object and a sort-of-action. '*A* is capable of *Y*-ing' is only superficially of the same form as '*A* loves *B*', with 'is capable of' in place of 'loves' and '*Y*-ing' in place of '*B*'. 'Is capable of' is not, to use a Polish technicality, that sort of functor. The real functor here is 'is capable of—ing', and what goes in the gap is not a noun but a verb. 'Is capable of —ing' is something like 'believes that someone —', where the gap could be filled by, say, 'smokes', or 'has magical powers'. In the strict sense of 'are', there 'are' no actions and no capacities, but things that act and things that are capable of acting. And, I repeat, it is *their* limitations—the limitations of actually present things, not those of still absent events—which, while leaving some alternatives possible, do not leave all alternatives possible.

We really have, I think, now driven away the spectre which Edwards has been dangling in front of us. But it is important to remember how we have done it. We have done it by falling back on a metaphysic of 'substances' endowed with capacities and dispositions, and our line of escape would hardly be open to someone (let us say Professor Hartshorne) who believes that objects are logical constructions out of events rather than vice versa. And it seems to me that this apparent dependence of the distinction between a limited and an unlimited indeterminism on this metaphysic of substances endowed with capacities, is a definite argument in favour of such a metaphysic. For the distinction between limited and unlimited indeterminism *is* one which it seems prima facie possible to make.

If we adopt a 'substance' metaphysic, for this or any other reason, we must of course do it properly, and be prepared to wear its further trimmings and trappings. Substance-talk, for example, is *tensed* talk. The use of the 'earlier' and 'later' relations in making temporal references belongs basically to the event-and-process language, and if it is made fundamental we get a world-picture of events arranged in an unchanging string. If events are logical constructions out of 'things acting', then '*A*'s hitting *B* is (tenselessly) a later event than *C*'s hitting *D*' means simply that it is, has been, or will be the case (and sooner or later it's 'has been') that (i) *A* is hitting *B* and (ii) it has been the case that *C* is hitting *D*.

We must note further that as regards the causeless starting-to-

be of things themselves, we have implicitly allowed Edwards's point to stand. I understand that there are astronomers, Hoyle for example, who believe that immense numbers of hydrogen atoms are all the time starting to exist all over the place, with a certain regularity in their distribution. It is no part of Hoyle's theory that this process is causeless, but I want to be more definite about this, and to say that if it *is* causeless, then what is alleged to happen is fantastic and incredible. If it is possible for objects—objects, now, which really *are* objects, 'substances endowed with capacities'—to start existing without a cause, then it is incredible that they should all turn out to be objects of the same sort, namely hydrogen atoms. The peculiar nature of hydrogen atoms cannot possibly be what makes such starting-to-exist possible for them but not for objects of any other sort; for hydrogen atoms do not have this nature until they are there to have it, i.e. until their starting-to-exist has already occurred. That is Edwards' argument, in fact; and here it does seem entirely cogent, leaving us with no alternatives but that either this starting-to-exist is caused (and a very strange sort of causation this must be; but that is another story), or it is a mis-categorization to treat hydrogen atoms as 'substances with capacities'.

VII

IDENTIFIABLE INDIVIDUALS

I WANT to examine some of the things that Professor N. L. Wilson says about the identity of individuals in his paper on 'Substances without Substrata';[1] and then I want to raise a few further problems of my own.

We can best begin from Wilson's 'simple little puzzle' about Caesar and Antony: 'What would the world be like if Julius Caesar had all the properties of Mark Antony and Mark Antony had all the properties of Julius Caesar?'[2] Wilson's own approach to an answer is indirect—he begins by telling us not what such a world would *be* like but what it would *look* like. 'Clearly the world would look exactly the same under our supposition.' But this assumes that the question 'What would such a world look like?' is a proper one; which it surely is not. For his answer to it is meaningless until he specifies *to whom* this supposed world would look as he says it would. It *would* look exactly the same to him or to me; but would it have looked the same to Caesar or to Antony? In fact Julius Caesar had the experiences of being called 'Julius Caesar', being murdered on the Ides of March, and so on, and these are very different experiences from being called 'Mark Antony', dallying on the Nile with Cleopatra, and so on; so I don't see how this alternative course of events could possibly have looked the same to Julius Caesar; or—using a similar line of argument—to Mark Antony. So I cannot agree that, as Wilson goes on to say, 'our attempt to describe a distinct possible world has produced just the same old world all over again'.[3] I am not, indeed, convinced that even a world which looked to *everyone* just as the actual one does would necessarily *be* the same world (since no one sees everything); but even putting this doubt aside, since the world mentioned *wouldn't* look to everyone as the actual world does,

[1] *Review of Metaphysics*, 12, 4 (June 1959), pp. 521–39.
[2] Ibid., p. 522. [3] Ibid., p. 523.

it wouldn't be the same even by Wilson's own standards (unless, indeed, he is a solipsist, and equates how the world is with how it looks to *him*).

Wilson then goes on[1] to consider a peculiarly perverse person who maintains that what has just been supposed is in fact the case, i.e. who seriously contends that in fact it *was* Antony, not Caesar, who was murdered on the Ides of March, etc., and Caesar, not Antony, who dallied on the Nile with Cleopatra. With regard to such a person, Wilson says that 'it would seem at least plausible to suppose' that he 'is really not guilty of historical error, but is using the words "Caesar" and "Antony" with significations we attach to "Antony" and "Caesar" respectively'.[2] I don't disagree with Wilson at this point, except that he seems to me not half emphatic enough. I would say that what he suggests is even more than 'plausible', and that there is nothing else we *can* suppose such a man to be doing unless it be flatly contradicting himself. For Wilson explicitly includes being called 'Julius Caesar' and 'Mark Antony' respectively among the properties which are supposed to be interchanged;[3] and while he doesn't say so, we can fairly assume that when he says 'called "Julius Caesar" ' he means 'called "Julius Caesar" *by most people*', and similarly with being called 'Mark Antony'. But it is impossible consistently to maintain that the man whom most of us call not 'Caesar' but 'Antony' is the man who really had the experience (among other experiences) of being called by most of us not 'Antony' but 'Caesar'. It really is absurd to say, 'It isn't the person we all call "Julius Caesar" that we all call "Julius Caesar", but it is rather the different person whom we don't call "Julius Caesar" but "Mark Antony" whom we call "Julius Caesar" '. So certainly the most charitable thing to think of a man who says or implies this is that he is using names in an idiosyncratic way.

But why is this ridiculous person brought into Wilson's story at all? Apparently the argument is that because such a man must at best be supposed to be using language in an odd and private way, and at worst contradicting himself, this is all that we can suppose about a man (myself, for example) who says that Caesar *could* have been named 'Antony', and Antony 'Caesar', and each had the other's properties. This sounds

[1] pp. 524–5. [2] p. 526. [3] p. 522.

deplorably like the following argument which was discussed (and trounced) by William of Ockham: 'I am going to sit down tomorrow, so God, whose thoughts are always true, thinks that I am going to; but I could have been not going to, so God could have been wrong.'[1] The obvious answer is that if I had not been going to sit down, God would not have thought I was going to. And analogously, if Caesar had been called not 'Caesar' but 'Antony', then 'Antony' and not 'Caesar' is what we would have called him, so that we would *not* under those circumstances have described the situation by saying (in the manner of Wilson's eccentric gentleman) 'The person we call "Antony" is not really Antony but Caesar'. Nevertheless the person we would in the imagined circumstances be calling 'Antony' would be the person whom in the actual circumstances we call 'Caesar', and in the actual circumstances the correct and only way to describe the imagined circumstance is as 'one in which it is not Antony but Caesar who is called "Antony" '.

I have, nevertheless, my own qualms about this supposed exchange. In the first place, there is quite certainly at least one property of Antony's which it makes no sense to suppose Caesar exchanging with the corresponding property of his own, namely the property of *being Antony*. For if we do attempt to include this property among those exchanged, and so suppose that all of Antony's antics and experiences characterize someone who *is Antony*, and similarly all of Caesar's characterize someone who *is Caesar*, this is indeed to suppose things to be exactly as they are.

Properties which *entail* being Caesar or being Antony, as the case may be, are also obviously to be exempted from the exchange if it is to be an exchange at all. For example, Antony had the complex property of 'dallying with Cleopatra, and not dallying with Cleopatra without being Antony'; clearly no one but Antony can be consistently supposed to have this property, though someone else can easily be supposed to have the simpler property of dallying with Cleopatra. But this is a comparatively trivial extension of my last point; I have more vexing worries.

As Wilson himself suggests, one way in which his question

[1] William of Ockham, *Tractatus de Praedestinatione et de Praescientia Dei et de Futuris Contingentibus*, ed. P. Boehner (Franciscan Institute, St. Bonaventure, N.Y., 1945), pp. 19 ff.

may be put is by asking whether there is a 'possible world', distinct from the actual one, in which Caesar has all of Antony's properties (with, of course, the exceptions just mentioned) and Antony all of Caesar's. It is clear that any such possible world must *contain* both Antony and Caesar, that is to say the actual Antony and Caesar; and it is here that our new troubles begin. When we talk about 'possible worlds' we frequently do so as if each such world were a complete and separate idea in the mind of God (or some such place); and when thinking of them in this way I find it difficult to believe that *any* merely possible world can contain individuals identifiable as our Julius Caesar and our Mark Antony. My objection here is not at all the Leibnizian one that Caesar is or is defined by the sum of his properties, so that any individual with different properties (including relational properties; and so any individual set in a different world) could not have been Caesar. On the contrary, I am away over on the other side of this fence; and it is just because Caesar *isn't* a property or collection of properties, that it is impossible as it were to detach his identity from the *Caesar that is* and attach it to a merely imaginary person in a merely imaginary world.

The feeling to which I have just given voice may be in some way misguided; but it is worth giving way to it sufficiently to let it drive us into considering a somewhat different way of talking about possible worlds, namely thus: We might say that a possible world is (i) one of the alternative possible future outcomes of the present actual state of affairs; or by a natural extension (ii) anything that *was* a possible world in the preceding sense, i.e. an outcome of some *past* state of affairs which *was* possible at the time, though it may by now have been excluded by what has actually taken place instead. Or finally, (iii) we may use the phrase for anything that constitutes a 'possible world' in sense (i) or (ii), together with its past, so that a possible world in this last sense is a *total course of events* which either is now possible or was possible once. And perhaps the more abstractly 'possible' worlds considered in the last paragraph may be comprehended in the present sort, inasmuch as, for any abstractly possible world *W*, if you wipe out *enough* of the actual past you will presumably reach a state of affairs of which *W* would have been a possible outcome. This would seem to be so, at all events, if going back far enough takes you to the creative *fiat* of God.

However it may be with this last speculation, 'possible worlds' in our second sense undoubtedly include some in which the actual Julius Caesar figures; namely, at the very least, all those which constitute alternative possible continuations of his actual life-story. And there is a possible world in which Julius Caesar is called 'Antony', i.e. Julius Caesar could have been called 'Antony'; since possible sequels to parts of his life include, for example, adoption by Antony's family. But of course this is only a very minor tinkering with the actual, and remains so even if we go on to suppose a different upbringing to have made rather a different man of him. Can we not go further and suppose Caesar to have had the whole of Antony's life, including being born to Antony's parents?

It is always a useful exercise (and one insufficiently practised by philosophers), when told that something was possible, i.e. could have happened, to ask '*When* was it possible?' '*When* could it have happened?' So if Caesar could have had different parents, when could he have had them? *After* his birth, indeed after his conception—indeed, *at or after* his conception—it was clearly *too late* for him to have had different parents. But why not before? Do not the possible worlds in which Caesar figures include alternative sequels to what happened before he existed, in which we have him *entering* the stage at a different point? My difficulty here is that *before* Caesar existed (whether we suppose his conception or some other event to constitute the start of his existence) there would seem to have been no individual identifiable as Caesar, i.e. the Caesar we are now discussing, who could have been the subject of this possibility.

But this line of argument, it may be objected, proves too much. For if, before Caesar existed, there was no individual identifiable as Caesar to be the subject of the possibility of being born to *Mark Antony's* actual parents, neither was there an individual identifiable as Caesar to be the subject of the possibility of being born to *Julius Caesar's* actual parents. So there cannot have been at that time any such possibility as that of Caesar's being born to these parents. Yet in due course this non-possible thing actually happened!

Let's repeat this paradox, so that it is quite clear what is being said. Julius Caesar, i.e. a certain now-identifiable individual, did at a certain time begin to exist. But before that time, the

possible outcomes of what was going on did not include the starting-to-exist of *this* individual. However, they did include the possibility that there should be *an* individual born to these parents, who would be called 'Caesar', would be murdered on the Ides of March, and so on; and this possibility was in fact realized when Caesar was born and underwent all these things.

Is this really so outrageous and unparalleled? Some of the schoolmen made a puzzle of the sentence *Equus tibi promittitur.*[1] There are two ways in which I may promise you a horse: I may promise you a particular horse, or I may just 'promise you a horse' without undertaking to let you have any horse in particular. In the former case, there is a horse-that-I-have-promised-you, but in the latter case there is none, though any horse whatever that I make over to you will constitute a fulfilment of my promise. I may even say as I hand it over, 'Here is the horse I promised you', but it is my handing it over in fulfilment of my promise that makes it that—it wasn't that when I promised (I could not truly say of this or any horse, 'This *was* the horse I promised you'); nor would it make sense for you to raise a doubt on the point and ask, 'But is *this* the horse you promised me?—are you sure it's *this* one?' With the other sort of promise, this question would of course be entirely in order. And what I want to say now is that the possibility that *an* individual should begin to exist and do and undergo such-and-such things, is like a promise of the second kind. *Any* individual's starting to exist and doing and undergoing the things in question will constitute a realization of this possibility; yet one cannot say of any individual that what was possible was that *he* should begin to exist and do and undergo these things; there just cannot be a possibility of that sort (which would correspond to a promise of the *first* kind) except with respect to what already exists (and so no possibility, of this sort, of existence itself).

Put it this way: Suppose there is some person living before the existence of Caesar or Antony who prophesies that there will begin to be a person who will be called 'Caesar', who will be murdered, etc., and another person who will be called 'Antony', who will dally with Cleopatra, etc. And then suppose this prophet to say, 'No, I'm not sure now that it *will* be like that—

[1] See, e.g., W. Burleigh, *De Puritate Artis Logicae Tractatus Longior*, ed. P. Boehner (Franciscan Institute, St. Bonaventure, N.Y., 1955), pp. 13 (*Dubium* 2), 14, 15.

perhaps it is the *second* of the people I mentioned who will be called "Caesar" and will be murdered, etc., and the first who will be born later and be called "Antony", etc.' This, it seems to me, really would be a spurious switch; and after Caesar and Antony had actually come into being and acted and suffered as prophesied, it would be quite senseless to ask 'Are these, I wonder, really the two people he meant?' and if possible more senseless still to ask, 'Is it—if either of them—our man's first prophecy, or his suggested alternative, that has now come to pass?' 'The (merely) possible', as Peirce said, 'is necessarily general', and 'it is only actuality, the force of existence, which bursts the fluidity of the general and produces a discrete unit'.[1]

Wilson, one feels at first, has read into our present condition, in which it really does mean something different to say that Caesar did and suffered this and that and to say that Antony did, the indeterminacy on this point which obtained before Caesar and Antony existed, when we could speak and think only of suppositious individuals. But one's second thought is that maybe Wilson is not all that far out, since in order fully to imagine an interchange of Antony's and Caesar's properties we must as it were pass through this Limbo in which their identities are lost, and then we can never regain them. Or, to drop the metaphor: in filling in the details of this supposed exchange, we must look back to the time before either Caesar or Antony existed, and as alternative futures to *this* time the two histories mentioned by Wilson really are indistinguishable.

But could not even God Himself have launched Julius Caesar into being, or arranged his coming into being, at a different time and under different circumstances? I doubt it; and I am not the first to have doubted it either. Thomas Aquinas doubted it; and though I have read what he has to say on this subject again and again, I am still not sure what his final opinion really amounts to. The relevant passages are in his *Quaestiones Disputatae de Potentia Dei*, Q. 3, *De Creatione*, Articles 1 to 3. The immediate subject here is the creation of the world out of nothing, but much that is said bears equally upon the bringing into being of any individual subject that was not there before. Like any good writer on this topic, Thomas insists that when we speak of *creatio ex nihilo*, we don't mean that 'nothing' was some sort of *material*,

[1] C. S. Peirce, *Collected Papers*, 4, 172. Cf. my *Time and Modality*, p. 114.

no doubt very tenuous, out of which the world was made. That the world was made out of nothing just means that it *wasn't made out of anything*; and indeed in this sense God was 'made out of nothing' too—there was nothing *He* was made out of, because He wasn't made at all. But, as Thomas nicely puts it, this manner of speaking is not usual, and the world was made out of nothing in the narrower sense of being indeed made, but not made out of anything.[1] But what was it of which it was possible, before it was made, that it should be made? Nothing, Aquinas says—it was possible that there should be a world, and God had the power to make a world, but there was nothing that had the power of being made.[2] But if creation was a gift of existence, he imagines an objector asking, *to what* was existence given? If to nothing, then nothing was created. If to something, then it must have had its own existence before the existence was given.[3] To this Aquinas answers that in creation 'God at the same time gives being and provides that which receives being';[4] an answer which I do not fully understand, but it is at least clear that Aquinas has *not* said, and has very deliberately not said, that the world somehow existed before it did exist in order to receive (and before that be capable of) existence.

'Is Creation a Change?' he asks next,[5] and answers that although the unreflective may regard it as the most radical change conceivable, it is not properly speaking a change at all. For in what is properly called change, one and the same subject must first have this or that true of it, and then not ('one same thing must be otherwise than it was before')—for example, if at one time X is $\mathcal{Z}$, and at another time Y is not $\mathcal{Z}$, there need be no change here ('two contraries if referred to different subjects can exist simultaneously'). But we do not have any one subject X first being $\mathcal{Z}$ and then not, or first not being $\mathcal{Z}$ and then being $\mathcal{Z}$, when X begins to be; for before it was, there was just no X. But is not this itself a change in the wider sense in which there is change if at one time P and not Q is true, at another time Q and not P? Thomas answers that there can be nothing like this with the creation of the world, since before the world there was no time. My own answer would be that 'Once X was not, and

[1] Art. 1, ad Obj. 7. [2] Art. 1, ad Obj. 2. [3] Art. 1, Obj. 17.
[4] Ad Obj. 17. [5] Art. 2.

now it is' cannot mean 'Once *X*'s non-being was the case and now its being is', but can only mean 'It *is not* the case that *X was*, but it *is* the case that *X is*', and this does not express a change but two contrasting present facts (note the tense of the two main verbs).[1]

In Article 3 Thomas goes further and says that being created is not properly speaking a *passion* in a thing, i.e. something that it has done to it, and insists that not only must a finite being *be created* in order to *be*, but any being must *be* in order to *be created*. ('This relation'—i.e. being created—'is an accident, and considered in its being, is subsequent to the thing created.')[2] So on Thomas's view, whatever else is obscure, it seems clear that there can be no question, even for God, of grabbing hold of *Caesar* and bringing him from nothingness to being at some arbitrary time. Expressions like 'launching' Caesar into being, and Caesar 'entering' upon the stage of existence, would certainly have been recognized by Thomas as the metaphors they are, and as misleading metaphors to the extent that their literal performance would involve Caesar's existing-before-he-existed. And this seems very close indeed to the admission that it is only once he exists that Caesar is an identifiable individual, and that God did not and could not 'create Caesar' in any sense in which, He having said 'Let there be a man, with properties *X* to *Z*', and there then starting to be a man with those properties, one could intelligibly ask 'But was *this* the man you wanted?' or intelligibly say 'This—this and no other—*was* the man God intended'.

To stop at this point, however, even with this remarkable theological reinforcement, would be to forget where we began. Let us leave both God and Caesar out of it for a moment; could not *I* have been born when and where Mark Antony was and had all his properties and experiences? As each of us puts this question to himself, it will seem clear that the thing is at least logically possible, i.e. not a self-contradictory supposition, and discernibly different from the actual state of affairs. We have already, in fact, in Section I, imagined Caesar putting this question to himself and having to give this answer. We may reflect also that if the theories of pre-existence and transmigration had been true, Caesar *would* have existed before his

[1] Cf. my *Time and Modality*, pp. 34–35.

[2] Art. 3, ad Obj. 3.

conception and so could have waited and been born to Mark Antony's parents at Mark Antony's time, and Mark Antony, also pre-existing, could have been born a little earlier to the people who in fact had Julius Caesar as their child; and we do not *know* that doctrines of pre-existence and transmigration are *not* true. Even if most of us regard it as so unlikely that belief in it is not a 'live option', it seems at least logically possible that it should be true.

I have no wish to maintain that we cannot be mistaken as to what is logically possible. On the contrary, to fall into error on this subject is extremely easy. For example, the following imagined sequence of events is one which most people will be inclined to regard as logically possible: A certain Mr. *X* has a very low opinion of the intelligence of a colleague, Mr. *Y*, and the two of them are walking down a corridor and eventually separate into adjacent rooms; Mr. *Y*, as he thinks, into Room 7, and Mr. *X*, as he thinks, into Room 8. It is towards 6 o'clock and Mr. *X* reflects for a while, as he often does, on the incurable stupidity of Mr. *Y*, and at precisely 6 o'clock the thought occurs to him that whatever is being thought at 6 o'clock that night in Room 7 is false. That is all that he thinks at that time, and there is no one else in the room with him. But unfortunately, owing to some inadvertence, it is he himself who has gone into Room 7, and Mr. *Y* is in fact in Room 6. Most readers who have followed me so far will, I think, have found this chain of events a possible one, however curious. Yet it is easy to demonstrate that what I have described is incompatible with the most elementary logical laws. For what Mr. *X* takes to be the case must either be the case or not be the case. And it cannot be the case, for if it were, i.e. if it were the case that nothing thought at 6 in Room 7 is the case, then it would not be the case, since it is itself thought at 6 in Room 7. And since it thus cannot be the case that nothing thought at 6 in Room 7 is the case, then the fact must be that *something* thought at 6 in Room 7 is the case. But this true thought in Room 7 cannot be the one we know about, since that, as we have just seen, is false. So there must be some other thought than this one occurring at 6 in Room 7; but our hypothesis was that it was the only one. Our hypothesis as a whole, therefore, is implicitly self-contradictory, exactly as the hypothesis that there is a barber who shaves all those people and only

those people who do not shave themselves is implicitly self-contradictory. Yet I must confess that I still feel, and I suspect that this is true of other people also, that there could be a Mr. *X* and Mr. *Y* behaving internally and externally exactly as I have said; so great is the force of certain 'logical illusions'.[1]

I doubt, however, whether it is necessary to bring in the hypothesis of 'logical illusion' to explain the apparent logical possibility of an exchange of properties between Antony and Caesar, or even between Antony and me. What *is* necessary is to distinguish logical possibility from the sort of possibility that we have been considering hitherto, and to get their relation straight. The distinction we want is made, for example, by the fifteenth-century scholastic philosopher Ferdinand of Cordova,[2] when describing a certain argument for the 'necessity' of correctly predicted future events. What is in question, he says, is not that 'logical necessity' which arises from a certain relation between a proposition's terms, but rather that 'truth which can no longer be prevented' which Aristotle was thought (by the propounders of the argument) to have required for a proposition to be counted 'already true'. If this distinction be kept clearly in view, it will be profitable to look again at the other horn of my dilemma about the *time* at which it was possible for Caesar to have had other parents. Once he was already born, I suggested, it was too late for him to have any chance of having had other parents. The 'necessity' here hinted at is of Ferdinand of Cordova's second sort; but the proposition that *this* person (the one we know as 'Julius Caesar') should have had those other persons (the ones who were in fact the parents of Mark Antony) as his parents, is certainly 'possible' in the sense of containing no internal inconsistency, and moreover is different in sense from the proposition that *that* person (the one we know as 'Mark Antony') had those persons as his parents.

In making this distinction, it is tempting to say that logical necessity and possibility are independent of the passage of time; and that about this sort of possibility the question '*When* was it possible?' need not and in fact cannot sensibly be asked. This,

[1] Cf. my 'Epimenides the Cretan,' *Journal of Symbolic Logic*, vol. 23, no. 3 (September 1958), pp. 261–6.

[2] In L. Baudry's, *La Querelle des Futurs Contingents* (J. Vrin, Paris. 1950), p. 139.

however, seems to be over-simplification. For there can be no truths, not even logical truths, that are distinguishably 'about' Caesar and Antony until there are such persons to be the subjects of these truths. Hence, while the passage of time may eliminate 'possibilities' in the sense of alternative outcomes of actual states of affairs, and cause that to be no longer alterable which once might have been otherwise, with 'logical' possibilities the opposite change occurs. For as new distinguishable individuals come into being, there is a multiplication of the number of different subjects to which our predications can be consistently attached, and so a multiplication of distinguishable logical possibilities. What was once just a possibility that 'someone' should have such-and-such a history, and 'someone else' should have such-and-such another history, can now be replaced by the distinct possibilities that X should have had the first history and Y the second, and that Y should have had the first and X the second. We must also accept the slightly odd result that it is logically possible that Julius Caesar *should have been* the son of Mark Antony's parents, even though before he existed it was not logically or in any other way possible that he should *come to have* those people, or any other people, as his parents. One must, in other words, take very seriously the present tense of the main verb in 'It is logically possible that Julius Caesar should have, etc.', just as one must take it very seriously (*v.s.* on creation and change) in 'It is not the case that Julius Caesar existed in 200 B.C.'

VIII

TIME, EXISTENCE, AND IDENTITY

FOR some years now a number of us have been working on what is called 'tense logic', in which an attempt is made to give something of the rigour of modern logical systems to a language whose sentences resemble those of natural languages in being, in some cases at least, true at one time and false at another. In such a language it is possible to have rudimentary tense-inflections, notably *Fp* for 'It will be the case that *p*', which is, of course, true whenever *p* itself is going to be true at some future time, and *Pp* for 'It has been the case that *p*', which is true whenever *p* has been true at some past time. The extensions of *propositional* calculus which we obtain by adding such modifiers to the usual *Np* ('It is not (now) the case that *p*'), *Cpq* ('If *p* then *q*'), *Apq* ('Either *p* or *q*'), *Kpq* ('Both *p* and *q*'), etc., are now fairly well understood, but we encounter serious problems when we attempt to introduce the same inflections into *predicate* logic, with quantification over individual variables.

To see where the difficulty lies, something should first be said about what these individual variables are supposed to stand for. In a logic with tenses, it is natural to let them stand for the 'things' of ordinary speech, that is, 'substances' in the old sense, or what W. E. Johnson calls 'continuants', objects such that we can say of each one of them that once it had such and such properties and did and suffered such and such things, that now it—the very same object—has such and such other properties and does and suffers such and such other things, and in the future it—the very same object—will have different properties again, and do and suffer different things. Tables and chairs and horses and men are typical 'individuals' of the sort intended; we may say of such-and-such a man, for example, that once he was a boy and now he is grown-up and some day he will be old, or that yesterday he was ill and now he is on the mend and tomorrow he will be quite better. And while in general these

individual objects have parts—men have arms and legs and so on—and these parts are themselves objects of a sort, we do *not* say that they have *temporal* parts or phases, in the way that processes and histories do. My boyhood, for example, is not a part of me, though it is a part of my history; and it is not the case that one part of me was a boy in New Zealand while another part of me is a man in England; it is I who was that boy, and I —the same I—who am the man.

This language is surely well enough understood, and most of the time fairly easily handled. When, however, we allow such 'substances' or 'continuants' to be the 'values' that the variables *x*, *y*, etc., can take in formulae like 'For some *x*, it was the case 40 years ago that *x* is a boy in New Zealand', we are soon compelled to take account, in one way or another, of the fact that most individuals of this sort have been in existence, and will be in existence, for a finite time only, and that the individuals in existence at any given time are not entirely the same as the individuals in existence at other times. We might, indeed, attempt to argue on physical or metaphysical grounds that tables and chairs and horses and men are not genuine individual continuants but only collections of these, the real continuants being certain ultimate 'simples' which exist throughout all time and merely get rearranged in various ways. Or we might argue that there is only a single genuine individual, the Universe, which gets John-Smithish or Mary-Brownish in such-and-such regions for such-and-such periods. But, prima facie at least, this is *not* how things are, and it ought to be at least possible to develop a tensed predicate logic which does not depend for its validity on any such assumptions. And, in fact, this is not only possible, but has been already done, in a variety of ways.

One very natural way of proceeding is to say, when we have a tense-operator such as 'It will be the case that' followed by a quantifier such as 'For some *x*', that the objects that are relevant to the verification of the quantified propositions are the ones which will be in existence at the time to which the tense-operator takes us. If I say, for example, that it will be the case that for some *x*, *x* is flying to the planet Mars, what I mean is that something *then* existing (whether it already exists now or not) will be flying to the planet Mars. If, on the other hand, we have a

quantifier right at the beginning of a sentence, we can suppose it to be governed by the operator 'It is the case that', i.e. 'It is now the case that', which can be prefixed to any tensed sentence without making any difference to what is said. So the form 'For some x, it will be the case that x is flying to the planet Mars' may be taken to mean 'Something which now exists will be flying to the planet Mars'.

That, at all events, is one way of handling this problem, and we now have a fair idea of what sort of tensed predicate logic we will get if we do it this way. We might argue, on the other hand, that it is just as easy to refer to what once existed, and perhaps just as easy to refer to what is going to exist, as to what exists now, and we might use *all* such objects, whether they exist now or not, as values of our variables, and if what we mean by 'Something' is 'Something that now exists' we must explicitly say so. For example, we may take both of the foregoing forms, namely,

'It will be the case that, for some x, x is flying to the moon'

and

'For some x, it will be the case that x is flying to the moon'

to mean merely that something that exists or has existed or will exist will fly to the moon, and the stronger form must be written as

'For some x, x now exists and it will be the case that x flies to the moon'.

This sort of tensed predicate logic is also fairly well developed now.

I have an uneasy feeling, however, that both these forms of tensed predicate logic operate with too crude and stark a notion of the beginning-to-be and ceasing-to-be of individuals. It is as if at one moment these individuals are just not there, and at another they are. In the real world it seldom seems as simple as this, although neither is it as simple as it would be if the only individuals were permanent 'ultimate simples', or if there were only a single genuine individual, also permanent. To quote what I have said elsewhere, 'Very roughly, it would seem that countable "things" are made or grow from bits of stuff, or from other countable "things", that are already there. The precise logic of

this process hasn't been worked out yet, and until it has been, it seems likely that any tensed predicate logic can only be provisional in character.'

I shall not attempt in this paper to do anything like filling this gap, but I want to say enough to indicate that not only what I have called 'stark starting to be', but also kinds of 'starting to be' which are much less 'stark', present serious logical problems. The obvious alternative to something's starting to be *without* antecedents is its starting to be *with* antecedents, and it is tempting to describe this as *one thing becoming another thing*. But what could this be, if it is neither a case of one thing simply ceasing to exist and a quite different thing starting to exist (maybe in roughly the same locality), nor a case of some one thing radically changing in its qualities or nature? It is not at all clear that there could be anything in between these two, but at least one kind of change which does not seem to fall under either description, and which is in any case worth investigating, would be *one thing becoming two things*. There do seem to be at least approximations to this in nature, e.g. the 'multiplication by division' of unicellular organisms, and still closer approximations to it seem to be easily imaginable, e.g. *conscious* organisms which divide in two and retain after division a clear memory of their undivided state.

If anything does, in fact, happen which could be reasonably described as one individual thing becoming two individual things, it is natural to ask whether this would be consistent with the generally accepted laws of the logic of identity. These are, primarily, the two laws which may be expressed as follows (using the form '*Ixy*' for 'x and y are one and the same individual thing'):

(1) Ixx,

(2) $CIxyC\phi x\phi y$.

The first states that anything is the same individual as itself; the second, that if x and y are one and the same individual thing, then if anything at all is true of x, that thing must be *ipso facto* true of y (we would be inclined to say its being true of x is its being true of y, since x is y). The second law is sometimes called 'Leibniz's law', or 'the indiscernibility of identicals'. From these

two, certain others easily follow. In the first place, ordinary commutation takes us from (2) to

(3) $C\phi xCIxy\phi y$.

If in this we let our 'ϕ-ing' be 'being identical with x', i.e. if we substitute $I'x$ for ϕ', we obtain

(4) $CIxxCIxyIyx$,

which by detachment of (1) yields

(5) $CIxyIyx$,

asserting that identity is symmetrical. Again, if in (3) we let our 'ϕ-ing' be 'z's being identical with', i.e. if we substitute Iz' for ϕ', we obtain

(6) $CIzxCIxyIzy$,

asserting that identity is transitive. And if in (2) we let our 'ϕ-ing' be 'being identical with z', i.e. if we substitute $I'z$ for ϕ', we obtain

(7) $CIxyCIxzIyz$,

asserting that if x is identical both with y and with z, then y is identical with z.

It formerly seemed to me that the basic laws (1) and (2) would be compatible with one thing's becoming two, if we were prepared to adopt a slightly non-standard tense-logic.[1] The difficulty which seemed to demand this was the following one: law (2) merely states that (i) if x *is now* the very same individual as y, then whatever now goes for x now goes for y. It does not state that (ii) if x *was once* the very same individual as y, then whatever now goes for x now goes for y. We could infer from (i), however, that (iii) if x *was once* the very same individual as y, then it *was once* the case that whatever went for x went for y, and in a metric tense-logic (with *Pnp* for 'It was the case the interval n ago that p', and *Fnp* for 'It will be the case the interval n hence that p') we could infer that (iv) if x *was once* the very same individual as y, then whatever *then* went for x, *then* went for y, too, or symbolically

(8) $CPnIxyCPn\phi xPn\phi y$.

[1] A. N. Prior, 'Opposite Number', *Review of Metaphysics* (1957).

Now let us suppose that the individual x is no longer identical with y (x and y being the two things which some one thing has become), and that x is now ϕ-ing. Then in any ordinary tense-logic it would follow that it was the case the interval n ago that it would be the case the interval n later that x is ϕ-ing, i.e. we have

(9) $C\phi xPnFn\phi x$.

From this, by (8), it follows that it was the case the interval n ago that it would be the case the interval n later that y is ϕ-ing, and from this and the converse of (9) it would follow that not only x but y is now ϕ-ing. But if x and y really have become distinct, it is perfectly possible that x should now be ϕ-ing and y not. To avoid the undesirable deduction, I suggested dropping (9), a suggestion which seemed plausible since there are independent reasons (connected with determinism) for questioning (9) anyway.

But this move, unfortunately, is not enough to prevent us from encountering other troubles. Let us suppose that the single individual x has become the two individuals y and z. If x has really *become* these two individuals, and has not simply ceased to exist and been in some sense replaced by them, then if anyone were to ask 'Where is x now?', one correct answer would be to say 'Here he is' and point to y. In other words, x is now y, and it would perhaps also be true to say that it is y who is now x, i.e. y is now x. But it would be equally correct to answer the question 'Where is x now?' by saying, 'Here he is', and pointing to z. In other words, x is now z; and, perhaps, z is now x. But from these premisses, the laws (6) and (7), above, would lead us to conclude that y is now z (this follows most straightforwardly, from Ixy and Ixz, by the use of (7)). But *ex hypothesi*, y is *not* now z. We are therefore led to question not merely the rather dubious tense-logical principle (9), but even the fundamental identity principles (6) and (7), and therefore, of course, the still more fundamental 'Leibniz's law' (2), from which (6) and (7) immediately follow.

This seems to me in some ways a much more serious objection to Leibniz's law than a number which have been offered in recent years. For these other objections generally depend on replacing the individual variables x and y by *descriptions*, and the

apparent breaches of the law can be eliminated by paraphrasing the descriptions away in the manner, say, of Russell. Instead of the examples normally used, drawn from modal logic or the logic of belief and knowing, we might here usefully consider a tense-logical example of substantially the same kind. Suppose that it is the case today that in a certain military unit the adjutant is the orderly officer (Ixy). It is always the case that the orderly officer is the orderly officer (ϕx, where ϕ— = 'It is always the case that the orderly officer is—'). But it does not follow that it is always the case that the orderly officer is the adjutant (ϕy). Here we simply have a couple of descriptions which sometimes apply to the same person and sometimes do not; what the first premiss really means is that someone is now both the adjutant and the orderly officer; the second premiss, in the only sense in which it is true, means that whoever is at any time the orderly officer is at that time the orderly officer; while in the only sense in which it yields the conclusion, the second premiss means that whoever is now the orderly officer is at all times the orderly officer, and in this sense it is false. What we do *not* have in this example is a couple of individuals who are sometimes the same individual and sometimes not. Similarly, no startling departures from ordinary views of identity are involved in the statement that 'the orderly officer is a different individual every day'.[1] What this means, of course, is simply that on each day a different person is-the-orderly-officer, i.e. answers to that description.

Nothing so simple as this, however, is involved in the example we began by considering. Here we are supposing that a certain two individuals *were* at one time one and the same individual, and the *descriptions under which* they are presented at the different periods are quite irrelevant. And it now seems to me quite clear that the only way in which the ordinary logic of identity can be fully preserved is by maintaining that cases of this sort never occur or can occur, i.e. that it never is or can be the case that one individual thing becomes two individual things; and that whenever we are tempted to describe an empirical change in this way, what has really happened has been the ceasing to be of one individual and the beginning to be of two others, or else they were two all the time, only this was not apparent, or else they

[1] I owe this example to E. J. Lemmon.

are still, in spite of appearances, one. This would be a reversion to the 'stark starting-to-be' which seems on the whole to be a rather *simpliste* notion, or to the theory that all genuine individuals are sempiternal.

Even if we are persuaded by these arguments that Leibniz's law is false, it remains obvious that something very like it is true, and we have to hunt for that. Here I content myself with two observations. In the first place, it may well turn out that the identity-functions for which we really do have a law like Leibniz's, and laws like *CIxyCIxzIyz*, are functions of higher order than identity of individuals. For example, if we have not a two-place predicate but a connective *I* such that *Ipq* may be read as 'The proposition that *p* is the very same proposition as the proposition that *q*', it may well be the case that for all *p*, *q*, and *r*, if *Ipq* and *Ipr* then *Iqr*. Secondly, even at the level of identity of individuals, it would seem that *x* will only be both the very same individual as *y* and the very same individual as *z*, if *y* and *z* at least *have been* one and the same individual, or maybe also if they are going to be (I have confined myself to the case of fission; fusion presents similar problems, and perhaps others as well). This means that we do have at least *CIxyCIxz AAIyzPIyzFIyz*.

We may, indeed, bypass all of these problems by simply dropping the notion of individual 'things' or 'continuants' which persist and act and interact as time goes on, in favour of the notion of four-dimensional objects which simply occupy so much time in the way that they occupy so much space; and we may suppose that some such objects have a branching structure in their temporal dimension. Or more precisely, we may suppose that the earlier temporal parts of a certain four-dimensional object are undivided spatially, while its later temporal parts are spatially divided. A rudimentary logic of the extension in time of 'objects' of this sort has been sketched by Carnap,[1] and something of the same sort is at present being developed by C. Lejewski. Lejewski's work is being done in the context of the Leśniewskian calculus, 'mereology', which deals with the part–whole relation between objects. On top of this he builds a discipline which he calls 'chronology', involving such conceptions as those of one object being wholly earlier than another,

[1] R. Carnap, *Introduction to Symbolic Logic*, Section 52.

and of one object having a greater temporal extension than another.

The development of this new body of theory is something which is only to be welcomed, but to leave the matter there, and regard our problem as solved by presenting its data in 'chronological' terms, is unsatisfactory in at least two ways. In the first place, the 'chronological' solution is only a new variant of the solution according to which the two objects remain one even after the division. It is based on the 'mereological' thesis that for any pair of objects there is a third object which simply consists of the two of them together, and this third object remains such whether its parts are continuously connected or separate. More precisely, each of the later contemporaneous slices of our four-dimensional object is as much a single object as each of its earlier slices, although the later ones have wholly distinct parts. There are no doubt senses of 'object' for which talk of this kind is appropriate, but they are not the senses in which the situation envisaged presents us with one object becoming two.

Secondly, and most seriously, to rest content with 'chronological' language as it stands would be to abandon the whole enterprise, which is one of the things that makes tensed predicate logic philosophically interesting, of exhibiting events and processes which are ordered and extended in time, and which can be conceived as having 'temporal parts', as logical constructions out of persisting and acting things or continuants. As was observed earlier, the successive phases of the history of a thing are in no sense parts of the thing itself; it is one and the same thing (the whole thing, so far as talk of parts and wholes is appropriate here) which at one time does or undergoes this and at another does or undergoes that, and at one time stands here and at another time—by which time it (the same thing) *has stood* here—stands there. It is within this framework that we must try to give the language of 'chronology' its meaning; not vice versa. This enterprise has been carried so far now that it is worth at least exploring the consequences of carrying it through to the end. But we cannot do this satisfactorily until much more work has been done, of the sort that is already beginning to be done, on the whole notion of an individual thing, and of 'the same individual thing'. If, even at this early stage, it appears

that we shall be faced with the abandonment or at least the modification of Leibniz's law, we may reflect that this has been contemplated in recent years for quite trivial reasons, so we need not be too dismayed if we now have to contemplate it for serious ones.

IX

STRATIFIED METRIC TENSE LOGIC

1. *Statement of the problem.* By a non-metric tense logic I mean propositional calculus enriched by the forms *Fp* for 'It will be the case that *p*', and *Pp* for 'It has been the case that *p*', with suitable postulates. By a metric tense logic I mean propositional calculus enriched by the forms *Fnp* for 'It will be the case the interval *n* hence that *p*' and *Pnp* for 'It was the case the interval *n* ago', with quantifiers binding interval-variables and manipulated according to the usual rules, and with suitable postulates for the special symbols. A certain amount of number-theory may be incorporated either in the metalogic or in the system itself.

Non-metric tense logic may be correlated with a theory of the earlier-later relation, in which the forms *Uab*, *Iab*, and *Tap* are used for 'The instant *a* is earlier than the instant *b*'; 'The instant *a* is identical with the instant *b*' and 'It is the case at the instant *a* that *p*' respectively, with the following postulates regarding the truth-conditions of tense-logical formulae:

U1. *ETaNpNTap* U2. *ETaCpqCTapTaq*
U3. *ETaFpΣbKUabTbp* U4. *ETaPpΣbKUbaTbp*,

and with or without special conditions (e.g. transitivity) on the relation *U*. E. J. Lemmon has shown that if no such special conditions are laid down, the tense-logical formulae which may be derived, preceded by *Ta*, from propositional calculus, quantification theory, and the postulates U1–U4, are precisely those which may be derived by substitution, detachment, and the rules

$$\vdash \alpha \rightarrow \vdash \mathit{NFN}\alpha,\ \vdash \mathit{NPN}\alpha$$

from propositional calculus and the axioms

1.1. *CNFNCpqCFpFq* 1.2. *CNPNCpqCPpPq*
2.1. *CFNPNpp* 2.2. *CPNFNpp*.

(The system defined by these postulates was called by Lemmon the system K_t.) It is useful to introduce the forms *Gp* ('It will always be the case that *p*') and *Hp* ('It has always been the case that *p*') as abbreviations for *NFNp* and *NPNp* respectively, turning the consequents of the rules into $\vdash G\alpha$ and $\vdash H\alpha$, and the axioms into *CGCpqCFpFq*, *CFHpp* and their mirror images. Additional tense-logical postulates may be correlated with special conditions on *U*, notably *CFFpFp* with transitiveness (*CUabCUbcUac*), *CFpFFp* with density (*CUabΣcKUacUcb*), *CNFpFNp* with non-ending, *CNPpPNp* with non-beginning, *CKFpFqAAFKpqFKpFqFKqFp* with non-branching in the future, and its image with non-branching in the past.

Postulates hitherto suggested for metric tense logic (e.g. by Rescher[1] and by myself[2]) do not have this 'stratified' character. For example, they include the rule $\vdash\alpha \rightarrow \vdash F_n\alpha$ and the axiom *CNFnpFnNp*, which assume non-endingness, the axiom *CFnNpNFnp*, which assumes non-branching in the future, and the axiom *CFmFnpF(m+n)p*, which assumes the transitivity of the earlier-later relation. What I shall attempt here is to give a postulate-set for metric tense logic which makes no such assumptions, with the additions required to bring various assumptions in. I shall also devise a correlated earlier-later calculus, analogous to the U-calculus which is correlated with non-metric tense logic.

2. *The metric U-calculus, and the minimal metric tense logic.* Both in the metric U-calculus and in the metric tense logic, the interval-variables *m*, *n*, etc. will be taken as standing for positive numbers measuring the intervals used, and as much number-theory as we need will be incorporated in the calculus. If the range of numbers drawn upon is the integers, the calculus will be suitable for time considered as discrete; if the rational numbers, for time considered as dense; if the reals, for time considered as strictly continuous. The number-theory drawn upon in what follows will require no more, in the way of symbols, than the forms *Imn* for '$m = n$' and *Smn* for '$m+n$'.

[1] In 'On the Logic of Chronological Propositions', *Mind*, vol. 75, no. 297 (Jan. 1966), pp. 75–96.

[2] e.g. in *Past, Present, and Future*, ch. vi.

In the U-calculus, in place of the non-metric *Uab* we shall use the form *Uabn* for 'The instant *a* is earlier than the instant *b* by the interval *n*', and replace the original U3 and U4 by the pair

U3. $ETaFnp\Sigma bKUabnTbp$
U4. $ETaPnp\Sigma bKUbanTbp.$

We also need a new postulate to cover quantification over intervals, namely

U5. $ETa\Sigma n\alpha\Sigma nTa\alpha,$

which is a schema covering the infinity of axioms in which α is replaced by any formula from the metric tense logic. We also need a postulate to make it clear that we regard an interval *n* as *added* to an interval *m* only if they are *contiguous* in the instant-series; i.e. we need

U6. $CUabSmn\Sigma cKUacmUcbn.$

These postulates suffice to yield, preceded by Ta, all theses derived from the following postulates for metric tense logic, subjoined to propositional calculus, quantification theory (with quantifiers binding interval-variables), and that part of positive number-theory which is common to the integers, the rationals, and the reals:

$$\vdash\alpha \rightarrow \vdash NFnN\alpha,\ \vdash NPnN\alpha;$$

A1.1. $CNFnNCpqCFnpFnq$	A1.2. $CNPnNCpqCPnpPnq$
A2.1. $CFnNPnNpp$	A2.2. $CPnNFnNpp$
A3.1. $CFm\Sigma nFnp\Sigma nFmFnp$	A3.2. $CPm\Sigma nPnp\Sigma nPmPnp$
A4.1. $CFm\Sigma nPnp\Sigma nFmPnp$	A4.2. $CPm\Sigma nFnp\Sigma nPmFnp$
A5.1. $CFSmnpFmFnp$	A5.2. $CPSmnpPmPnp.$

It will suffice to show this for the left-hand columns. In the first place,

$\vdash Ta\alpha \rightarrow \vdash Tb\alpha$	(substitution; α not affected, as it contains no *a*)
$\rightarrow \vdash CUabnTb\alpha$	(by $CpCqp$)
$\rightarrow \vdash \Pi bCUabnTb\alpha$	(by U.G.)
$\rightarrow \vdash N\Sigma bKUabnNTb\alpha$	
$\rightarrow \vdash N\Sigma bKUabnTbN\alpha$	(by U1)
$\rightarrow \vdash NTaFnN\alpha$	(by U3)
$\rightarrow \vdash TaNFnN\alpha$	(by U1)

Then for A1.1, note first that Ta(A1.1) expands by U1–4 to

$$CN\Sigma bKUabnTbNCpqC\Sigma cKUacnTcp\Sigma dKUadnTdq,$$

which is in turn equivalent to

$$\Pi cC\Pi bCUabnCTbpTbqCUacnCTcp\Sigma dKUadTdq,$$

which we prove thus:

ΠcC	(1)	$\Pi bCUabnCTbpTbq$	
C	(2)	$Uacn$	
C	(3)	Tcp	
K	(4)	$CUacnCTcpTcq$	(1, U1)
K	(5)	Tcq	(4, 2, 3)
	(6)	$\Sigma dKUadnTdq$	(2, 5, E.G.).

Ta(A2.1) expands to

$$C\Sigma bKUabnN\Sigma cKUcbnNTcpTap,$$

which equates to

$$\Pi bCUabnC\Pi cCUcbnTcpTap,$$

which we prove thus:

ΠbC	(1)	$Uabn$	
C	(2)	$\Pi cCUcbnTcp$	
	(3)	Tap	(1, 2).

Ta(A3.1) expands to

$$C\Sigma bKUabmTb\Sigma nFnp\Sigma n\Sigma cKUacm\Sigma dKUcdnTdp,$$

in which the initial $C\Sigma b$ is equivalent to ΠbC, and $Tb\Sigma nFnp$ to $\Sigma nTbFnp$ (by U5), and so to $\Sigma n\Sigma eKUbenTep$, where the Σn and Σe (introducing an antecedent) may be replaced by Πn and Πe at the beginning of the whole, which we may then prove thus (A4.1 being proved similarly):

$\Pi b\Pi e\Pi nC$	(1)	$Uabm$	
C	(2)	$Uben$	
C	(3)	Tep	
K	(4)	$\Sigma dKUbdnTdp$	(2, 3, E.G.)
	(5)	$\Sigma cKUacm\Sigma dKUbdnTdp$	(1, 4, E.G.).

Ta(A5.1) expands to

$$C\Sigma bKUabSmnTbp\Sigma cKUacm\Sigma dKUcdnTdp,$$

which we prove thus:

ΠbC	(1)	$UabSmn$	
C	(2)	Tbp	
ΣcK	(3)	$Uacm$	(1, U6)
K	(4)	$Ucbn$	(1, U6)
	(5)	$\Sigma dKUcdnTdp$	(4, 2, E.I.).

Whether the above *Fn–Pn* postulates suffice to yield *all* the metric tense-logical theses that are obtainable, preceded by *Ta*, from our minimal metric U-calculus, has not been proved: I offer it as a conjecture that they do. They certainly suffice to yield Lemmon's minimal non-metric tense logic K_t, if we introduce the forms $F\alpha$ and $P\alpha$ as abbreviations for $\Sigma nFn\alpha$ and $\Sigma nPn\alpha$, where n does not occur free in α. Indeed, we can prove this much without drawing upon the A3's and A4's. We prove the rule to infer $\vdash NFN\alpha$ from $\vdash\alpha$ by taking an n which does not occur free in α and proceeding thus:

$$\begin{aligned}\vdash\alpha &\rightarrow \vdash NFnN\alpha\\ &\rightarrow \vdash \Pi nNFnN\alpha \text{ (by U.G.)}\\ &\rightarrow \vdash N\Sigma nFnN\alpha\\ &\rightarrow \vdash NFN\alpha \quad \text{(by Df.}F\text{)}.\end{aligned}$$

And $CNFNCpqCFpFq$ thus:

(1)	$CNFnNCpqCFnpFnq$	(A1.1)
(2)	$C\Pi nNFnNCpqC\Sigma nFnp\Sigma nFnq$	(1, quantification theory)
(3)	$CN\Sigma nFnNCpqC\Sigma nFnp\Sigma nFnq$	(2)
(4)	$CNFNCpqCFpFq$	(3, Df.*F*).

And $CFNPNpp$ thus:

(1)	$C\Pi nNPnNpNPnNp$	(*Cpp* subst., $\Pi 1n$)
(2)	$CN\Sigma nPnNpNPnNp$	(1)
(3)	$NFnNCN\Sigma nPnNpNPnNp$	(2, Rule)
(4)	$CFnN\Sigma nPnNpFnNPnNp$	(3, A1.1)
(5)	$CFnN\Sigma nPnNpp$	(4, A2.1)
(6)	$C\Sigma nFnN\Sigma nPnNpp$	(5, $\Sigma 1n$)
(7)	$CFNPNpp$	(6, Df.*F*, Df.*P*).

The A3's, A4's, and A5's do not appear to yield, on their own, any further theses in non-metric tense logic, but they contribute to proofs in richer systems with axioms corresponding to special conditions on U. Before considering such systems, it is worth noting that the converses of the A3's are easily provable; that of A3.1 as follows:

(1) $CFnp\Sigma nFnp$ (Cpp, subst., $\Sigma 2n$)
(2) $NFmNCFnp\Sigma nFnp$ (1, Rule)
(3) $CFmFnpFm\Sigma nFnp$ (2, A1.1)
(4) $C\Sigma nFmFnpFm\Sigma nFnp$ (3, $\Sigma 1n$).

We may also note that the procedure by which we just obtained (3) from (1) via (2) may be generalized, i.e. we may use the rule and A1.1 in succession to pass from $\vdash C\alpha\beta$ to $\vdash CFn\alpha Fn\beta$; similarly with Pn. We may call these derived rules RFC and RPC.

It should also be noted, before we pass on, that in the minimal system Gp ($= NFNp$) is not demonstrably equivalent to $\Pi nFnp$. The definition of Π expands the latter to $N\Sigma nNFnp$, while the definition of F expands the former to $N\Sigma nFnNp$; these can only be equated when the minimal system is supplemented by certain theses discussed in the next section. Gp ($= N\Sigma nFnNp$) is, however, equivalent even in the minimal system to $\Pi nNFnNp$.

3. *Metric calculi reflecting special conditions on U.* We obtain richer calculi either by enriching the number-theoretical basis or by enriching the tense-logical postulates. To illustrate the first possibility, consider the effect of letting our intervals be measured by the real numbers, for which we have, for any number n, the law $\Sigma l\Sigma mInSlm$. We may then carry though the following proof:

(1) $CInSlmCFnpFSlmp$ ($CImnCflmfln$)
(2) $C\Sigma l\Sigma mInSlmCFnp\Sigma l\Sigma mFSlmp$ (1, quantification theory)
(3) $CFnp\Sigma l\Sigma mFSlmp$ (2, law for rationals).

From this we may deduce $CFpFFp$ (one of the theses generally used in non-metric tense logic to express the density of the earlier-later series) thus:

(4) $C\Sigma nFnp\Sigma l\Sigma mFlFmp$ (3, $\Sigma 1n$, A5.1)
(5) $C\Sigma nFnp\Sigma lFl\Sigma mFmp$ (4, $C\Sigma mFnFmpFn\Sigma mFmp$)
(6) $CFpFFp$ (5, Df.F).

Note the dependence of this proof on A5.1 as well as on the special postulate for density.

The converse of A5.1, namely *CFmFnpFSmnp*, is not provable in the minimal metric system as the converse of A3.1 is, nor does it have the same 'neutrality'. It yields, in non-metric tense logic, the thesis *CFFpFp*, which expresses the transitivity of the earlier-later relation, and its own U-calculus derivation, preceded by *Ta*, depends on this 'law of summation' for the triadic *U*:

CUabmCUbcnUacSmn

The following point is also to be noted about this thesis: if *CFFpFp* is added to the minimal non-metric system K_t we may derive its own mirror image *CPPpPp*, and vice versa. (Similarly with the density theses *CFpFFp* and *CPpPPp*.) The same is true of the metric thesis *CFmFnpFSmnp*, added to the minimal metric system. To shorten the proof of *CPmPnpPSmnp* from this thesis, we may first list some theses which are easily derivable in the minimal system, by various transformations and substitutions, from A2.1 and 2.2:

T1.1. *CpNFnNPnp* T1.2. *CpNPnNFnp*
T2.1. *CFnNPnpNp* T2.2. *CPnNFnpNp*.

We now give the main derivation:

(1) *CFmFnpFSmnp*
(2) *CNFSmnpNFmFnp* (1, p.c.)
(3) *CPmNFSmnpPmNFmFnp* (2, RPC)
(4) *CPmNFSmnpNFnp* (3, T2.2)
(5) *CPnPmNFSmnpPnNFnp* (4, RPC)
(6) *CPnPmNFSmnpNp* (5, T2.2)
(7) *CPnPmNFSmnNPSmnpPSmnp* (6, subst., *CNNpp*)
(8) *CPnPmpPSmnp* (7; T1.1, RPC).

(An analogous proof renders A5.2 superfluous as an axiom.)

For time's forward infinity we simply add *CNFnpFnNp*, from which the non-metric *CNFpFNp* is easily derivable. Time's backward infinity, *CNPnpPnNp*, does not of course follow, but must be laid down separately, if desired.

The converse thesis *CFnNpNFnp* is the expression in metric tense logic of time's non-branching in the future; it corresponds to the U-condition *CUabnCUacnIbc*. Non-branching in the past,

corresponding to *CUbanCUcanIbc*, is expressed by *CPnNpNPnp*. Neither of these is derivable from the other, but we may concentrate on the one in *F*. Among the theses which become derivable when we add *CFnNpNFnp* to the minimal metric system are

T1. *CFnpNFnNp*
T2. *CPnFnpp*
T3. *CPnFSnmpFmp*
T4. *CFnpCFnqFnKpq*
T5. *CPFpAApPpFp*
T6. *CKFpFqAAFKpqFKpFqFKqFp*.

The intuitive connexion between each of these theses and non-branching in the future is straightforward; we can see how each of them could have exceptions if there were no such thing as 'the' future but only a number of alternative possible futures. As a specimen, we prove T5, as follows:

(1)	*CImnCPmFnpPnFnp*	
(2)	*CImnCPmFnpp*	(1, T2)
(3)	*CImnCPmFnpAApPpFp*	(2, p.c.)
(4)	*CImSlnCPmFnpPSlnFnp*	
(5)	*CImSlnCPmFnpPlPnFnp*	(4, A5.2)
(6)	*CImSlnCPmFnpPlp*	(5, T2)
(7)	*CΣlImSlnCPmFnpΣlPlp*	(6, quantification theory)
(8)	*CΣlImSlnCPmFnpPp*	(7, Df.*P*)
(9)	*CΣlImSlnCPmFnpAApPpFp*	(8, p.c.)
(10)	*CInSmlCPmFnpPmFSmlp*	
(11)	*CInSmlCPmFnpPmFmFlp*	(10, A5.1)
(12)	*CInSmlCPmFnpFlp*	(11, T2)
(13)	*CΣlInSmlCPmFnpΣlFlp*	(12)
(14)	*CΣlInSmICPmFnpFp*	(13, Df.*F*).
(15)	*CΣlInSmlCPmFnpAApPpFp*	(14, p.c.)
(16)	*AAImnΣlImSlnΣlInSml*	(number-theory)
(17)	*CPmFnpAApPpFp*	(16, 3, 9, 15)
(18)	*CΣmΣnPmFnpAApPpFp*	(17, *Σ*1*n*, *Σ*1*m*)
(19)	*CΣmPmΣnFnpAApPpFp*	(18, A4.2)
(20)	*CPFpAApPpFp*	(19, Df.*P*, Df.*F*).

4. *Comparisons with earlier systems.* Given the additions for infinity, we may abridge the A1's to *CFnCpqCFnpFnq* and its image, and

derive the rules to infer $\vdash Fn\alpha$, $\vdash Pn\alpha$ from $\vdash \alpha$. (We then get our original A1.1 by $NFnNCpq \to FnNNCpq \to FnCpq \to CFnpFnq$.) Given the additions for non-branching, we may make the shorter rules (to infer $\vdash Fn\alpha$ and $\vdash Pn\alpha$ from $\vdash \alpha$) the primitive ones, and replace the A2's by the shorter T2 and its image. These abridgements are the normal versions of these postulates, but their adoption from the outset makes it difficult to see what depends on what, and impossible to devise systems for those who find some of the assumptions listed in the last section unpalatable.

In my earlier systems, a subtraction sign was introduced, the forms $F(m—n)p$ and $P(m—n)p$ being considered well-formed in those cases in which $m > n$, and such theses as $CFmPnpF(m—n)p$ being laid down for those cases in which they are well-formed. Such 'conditional well-formedness' is, however, an awkward notion, and in the present systems such notation is avoided. We can, however, define '$m > n$' as $\Sigma lImSln$, and express the above thesis, without any attached proviso to guarantee well-formedness, as

$$CImSlnCFmPnpFlp,$$

which is provable in any system that contains $CFnPnpp$. A minus sign is really only worth having in a system like Rescher's, in which intervals are represented by negative as well as positive numbers and Pnp in effect defined as $F(—n)p$, but such an economy is only possible in a fairly strong system, and removes all possibility of the stratification here attempted.

The present system for 'alternative futures', i.e. the system without $CFnNpNFnp$, is different from the 'Peircean' indeterminist metric tense logic which I have sketched elsewhere,[1] though each may be developed within the other. Broadly, the Peircean 'will' means 'necessarily will', whereas the present 'will' means 'possibly will'; the present Fn is definable in the Peircean system as $NFnN$—it is, in fact, the Mn so defined in *Past, Present, and Future*, p. 132, and the MFn so defined in Paper III—and vice versa. Hence, for example, the absence of $CFnNpNFnp$ from the present system corresponds to the absence of its converse from the Peircean. As far as indeterminism

[1] In Paper III, and more fully in 'Postulates for Tense-logic' (*American Philosophical Quarterly*, April 1966) and in *Past, Present, and Future*, ch. vii.

goes there is little to choose between the two versions of it, but the present set-up yields more natural ways of questioning, e.g. time's infinity. We would, all the same, get very slightly neater postulates all round if we used *Gn* and *Hn* for our present *NFnN* and *NPnN*, but made the former primitive and defined *Fn* and *Pn* as *NGnN* and *NHnN*.

The minimal system would then be:

$$\vdash \alpha \rightarrow \vdash G n \alpha, \vdash H n \alpha$$

B1.1 *CGnCpqCGnpGnq*	B1.2 *CHnCpqCHnpHnq*
B2.1 *CNGnNHnpp*	B2.2 *CNHnNGnpp*
B3.1 *CΠnGmGnpGmΠnGnp*	B3.2 *CΠnHmHnpHmΠnHnp*
B4.1 *CΠnGmHnpGmΠnHnp*	B4.2 *CΠnHmGnpHmΠnGnp*
B5.1 *CGmGnpGSmnp*	B5.2 *CHmHnpHSmnp*.

For transitivity we would add *CGSmnpGmGnp*, for infinity *CGnNpNGnp* and its image, and for non-branching *CNGnpGnNp* and its image. The system without *CNGnpGnNp* (but with its image) would then be precisely the Peircean, with *Gn* for the Peircean *Fn*.

X

THE LOGIC OF ENDING TIME

IT is sometimes said that not only the universe but time itself will come to an end; and it is perhaps more often said that not only the universe but time itself had a beginning. I intend to concentrate on the first and perhaps less likely of these two hypotheses, but what I have to say will be easy to apply, with appropriate modifications, to the other one. It is sometimes suggested that the idea that time will come to an end is essentially incoherent; and so it would be if we had to express this supposition by saying that at all times after a certain time there will be no time at all. We do not, however, have to put it this way, and I shall begin by developing a perfectly consistent logical system embodying the assertion, in a quite clear way, that time will have an end. It is also sometimes said, contrariwise, that the supposition that there will be an end to change but not to time is incoherent, and that to suppose that after a certain time the total state of the universe will remain for ever unaltered, differs no more than verbally from the supposition that time will have an end. I shall look at this contention, and try and see if there is anything in it, in the later part of the paper.

1. *Lemmon's minimal tense logic* K_t, *and the associated logic of the earlier-later relation.* In developing a form of tense logic which embodies the belief that time will have an end, we shall naturally have to avoid postulates which imply that time will *not* have an end. Some systems of tense logic, for example, have as an axiom 'It will be the case that if p then p', and since axioms are taken to be true at whatever time there is, this one would mean that there will always be a future, and so that time will *not* end. 'It will *always* be the case that if p then p', it should be observed, need not commit us to this. If we use (as we shall be doing here) what might be called a *Boolean* 'always', i.e. if we

so read 'It will always be true that—' as to make it equivalent to 'It will never be false that—', then 'It will always be that if p then p' doesn't commit us to there being any future; it only commits us to there being no future in which it is not the case that if p then p.

We can be sure of not having 'It will be the case that if p then p' either as an axiom or a theorem, if we begin from a system which E. J. Lemmon discovered in 1965 to be 'minimal' in a sense which we shall shortly define, and which he called the system K_t. In this system G (for 'it always will be that') and H (for 'it always has been that') are taken as primitive, and F (for 'it will be that') and P (for 'it has been that') defined respectively as NGN and NHN. We have the rules to infer $\vdash G\alpha$ and $\vdash H\alpha$ from $\vdash \alpha$ (RG and RH), and the following four axioms:

A1.1. $CGCpqCGpGq$ A1.2. $CHCpqCHpHq$
A2.1. $CPGpp$ A2.2. $CFHpp$.

Among the theorems obtainable from this basis are $CGCpqCFpFq$, $CKGpFqFKpq$, $EKGpGqGKpq$, $EFApqAFpFq$, $CpGPp$, and their images, and we may also obtain the rules to infer $\vdash CG\alpha G\beta$, $\vdash CH\alpha H\beta$, $\vdash CF\alpha F\beta$, and $\vdash CP\alpha P\beta$ from $\vdash C\alpha\beta$ (RGC, RHC, RFC, and RPC).

This system is 'minimal' in the following sense: an alternative method of formalizing facts about time is to use the variables a, b, c, etc. for instants, the forms Tap for 'It is the case at the instant a that p', Iab for 'The instant a is the same instant as the instant b', Uab for 'The instant a is earlier than the instant b', and to lay down the following four axioms for U and T (with the usual laws for I):

U1. $ETaNpNTap$ U2. $ETaCpqCTapTaq$
U3. $ETaGpΠbCUabTbp$ U4. $ETaHpΠbCUbaTbp$.

These assert respectively that (i) it is true at a that not-p if and only if it is not true at a that p; (ii) it is true at a that p-implies-q, if and only if p's being true at a implies q's being true at a; (iii) it is true at a that it will always be the case that p, if and only if p itself is true at all instants later than a; and (iv) it is true at a that it has always been the case that p, if and only if p itself is true at all instants earlier than a. If we subjoin these

postulates alone to propositional calculus and quantification theory (without adding any special conditions on U, e.g. that it is transitive), we can prove various tense-logical theses preceded by *Ta*, i.e. we can prove that various tense-logical formulae are true at any arbitrarily chosen instant *a*. For example, we can prove *TaCFKpqFp*, 'It is true at any instant *a* that if it will be the case that both *p* and *q*, it will be the case that *p*'. What Lemmon has shown is that the tense-logical theses which are thus provable (preceded by *Ta*) in this 'minimal U-calculus', are precisely the theorems of the tense-logical calculus K_t. If special conditions are put on the relation U, further tense-logical theses become deducible, e.g. if we lay it down that U is transitive, *CUabCUbcUac*, we can prove *TaCGpGGp*.

Proofs of tense-logical theses (preceded by *Ta*) in U-calculi are a fairly straightforward matter. But if certain further ideas and postulates are introduced, it is equally possible to prove U-calculus theses from tense-logical ones, e.g. *CUabCUbcUac* from the system K_t with the added axiom *CGpGGp*. The essential trick is to treat the instant variables as a special sort of *propositional* variables, by identifying an 'instant' with the totality of what would be ordinarily said to be true *at* that instant, or indeed by identifying it with *any* proposition which would ordinarily be said to be true at that instant only. Formally, we begin from some tense logic, say K_t, and introduce (i) an operator L, which means in effect 'It is true at all times that', and for which we lay down the modal system S5 plus the axioms *CLpGp* and *CLpHp*; and (ii) the set of variables *a*, *b*, *c*, etc. to stand for such propositions as satisfy the following axioms:

A1. *Ma*
A2. *ALCapLCaNp*
A3. *Σaa*.

Ma (= *NLNa*) asserts in effect that the proposition *a* is true at some time; A2 that *a* is 'comprehensive' in the sense that for any proposition *p*, either *a* permanently implies that *p* or it permanently implies that not *p*; and A3, that for some 'world-state-proposition' *a*, that proposition is true right now. (The rule of necessitation from S5 enables us to pass from this to the theorem that at any time some such proposition is true.) Finally, we

define *Tap* as *LCap* ('It is the case at *a* that *p*' = 'The world state-proposition *a* permanently implies that *p*', i.e. *p* is one of the propositions 'in' that particular totality); *Iab* as *LEab*; and *Uab* as *TaFb* ('Instant *a* is earlier than instant *b*' = 'It is true at *a* that it will be the case that *b*').

If our tense-logical basis is K_t it is possible, given the enrichments just mentioned, to prove the minimal U-calculus postulates U1–U4,[1] and with appropriate additions to our tense logic we can prove the corresponding conditions on the earlier-later relation *U*. I shall not attempt even to sketch a proof of this here, but one or two observations on the 'world' calculus may be in order. In the system as given, the operator *L* is not a tense-logical one, but on any view of time except the view that there exist several totally unconnected 'time-streams', *L* is tense-logically definable, and on some views of time quite simply so, e.g. if time is infinite and non-branching in both directions, and the earlier-later relation is transitive, *L* is equivalent to *GH* or *HG* ('It is true at all times that *p*' = 'It has always been that it will always be that *p*', or vice versa). In any case, the basis given enables us to prove for *L* such theses as *CLpLGp*, *CLpGLp*, *CFLpLp*, and *CPLpLp*. It does not, however, enable us to prove *CGLpLp*, i.e. 'If it will always be that *p*-at-all-times, then *p*-at-all-times', i.e. 'If it will never be false that *p*-at-all-times, then *p*-at-all-times'. And this is just as well, for if time will have an end, then *at* time's end the antecedent 'It will never be false that *p*-at-all-times' will be vacuously true for any *p* (for by then nothing at all is ever *going to be* false *or* true), but the consequent *p*-at-all-times is certainly not itself vacuously true for any *p*, even at the end of time.

Other easily provable theorems in the 'world' calculus include *CKapLCap*, or *CKapTap*, 'If the world-proposition *a* and the proposition *p* are simultaneously true, then *p* is one of the propositions which the world-proposition *a* permanently implies', i.e. one of the propositions true 'in' that world or 'at' that 'instant'; *CKaPaFa*, 'If the present total world-state is one that we have had before, it is one that we shall have again'; *CTabIab*, i.e. the only world-state proposition true at *a* is *a* itself; and *EΠaTapLp*, which could be used to define *L* if we did not

[1] Proofs of this, and of most of the other unproved assertions in this section, may be found in *Past, Present, and Future*, ch. v, § 6, and App. B, § 3.

use the latter to define T. (Our actual procedure—taking L as primitive instead of T—is only worth adopting if we regard L as being tense-logically definable.) These particular theses require only K_t for the underlying tense logic. For instance, we prove *CKapTap* thus:

C	(1)	*a*	
C	(2)	*p*	
K	(3)	*MKap*	(1, 2, *CpMp*)
K	(4)	*NLCaNp*	(3, *EMKpqNLCpNq*)
	(5)	*LCap*	(4, A2),

and *CKaPaFa* thus:

C	(1)	*a*	
C	(2)	*Pa*	
K	(3)	*HFa*	(1, *CpHFp*)
K	(4)	*PKaFa*	(2, 3, *CKHpPqPKpq*)
K	(5)	*PLCaFa*	(4, previous theorem, RPC)
K	(6)	*LCaFa*	(5, *CPLpLp*)
	(7)	*Fa*	(1, 6).

We may note also that the forms *Tap*, *Iab*, and *Uab* could have been introduced as special cases of the forms *Tpq*, *Ipq*, and *Upq*, similarly defined. But if we did this, it might be better to define *U* a little differently. If we read *Uab* as 'The instant *a* is earlier than the instant *b*' it is perhaps natural to read the more general *Upq* as 'The *event p* is earlier than the event *q*'. McTaggart plausibly suggested[1] that 'The term P is earlier than the term Q if it is *ever* past while Q is present, or present while Q is future', i.e. if there is *any* time at which we have P past and Q present, or P present and Q future; and this goes into symbols as defining *Upq*, not as *LCqPp* or *LCpFq*, but rather as *MKPpq* or *MKpFq*. The alternatives in each case are easy to prove equivalent; for example, we get *MKPpq* from *MKpFq* thus:

C	(1)	*MKpFq*	
K	(2)	*MKGPpFq*	(1, *CpGPp*)
K	(3)	*MFKPpq*	(2, *CKGpFqFKpq*, RMC)
	(4)	*MKPpq*	(3, *CMFpMp*),

[1] J. M. E. McTaggart, *The Nature of Existence* (Cambridge, 1927), § 610.

and the reverse by a similar use of *CpHFp*. And if we do define *Upq* as *MKpFq*, this can be proved equivalent in the case of 'worlds' to our original definition, thus:

C (1) *MKaFb*
K (2) *MLCaFb* (1, *CKapLCap*, RMC)
(3) *LCaFb* (2, *CMLpLp*).

And thus (for the converse):

C (1) *LCaFb*
K (2) *LCaKaFb* (1, *CCpqCpKpq*, RLC)
(3) *MKaFb* (2, A1, *CLCpqCMpMq*).

2. *Postulates for ending time.* One way of setting up a minimal logic for ending time is to add to the system K_t the one axiom *ANFpFNFp*, 'Either it *is already* the case that *p* will never be true, or it *will* be the case that *p* will never be true'. (Making this a thesis means, of course, laying it down as true at all times and for any *p*.) The first disjunct of this, it will be noticed, is something which is bound to be the case at the end of time; while the second is bound to be the case if the end of time is still ahead of us; so that if there *is* to be an end of time, the whole disjunction will be true throughout the whole of time (though neither it nor anything else will be true *after* the end of time. Indeed, 'after the end of time' is a self-contradictory description; but talk of time's having an end means that at a certain time, namely the last instant, all assertions to the effect that something *will be* the case, are false).

The postulate *ANFpFNFp* is equivalent to *CFpFNFp*. In itself this simply asserts that if anything will be the case, then it will sooner or later be the case that *that* thing will never be the case. But we may deduce from it the apparently more general thesis *CFpFNFq*, asserting that if anything will be the case, then it will sooner or later be the case, not only with respect to that thing, but with respect to anything you please, that it will never be the case. The proof is as follows:

1. *CFpFCpp* (*CpCpp*, RFC)
2. *CFpFNFCpp* (1, *CFpFNFp*)
3. *CFqFCpp* (*CqCpp*, RFC)

4. *CNFCppNFq* (3)
5. *CFNFCppFNFq* (4, RFC)
6. *CFpFNFq* (2, 5).

6 in turn easily equates to *CFpFGq*.

Given this result, we may prove a further one in that fragment of this tense logic which constitutes a Diodorean modal logic. Diodorus defined 'Possibly *p*', *Mp*, as 'It either is or will be that *p*', *ApFp*, and 'Necessarily *p*', *Lp*, as 'It is and always will be that *p*', *KpLp*. (These are of course a different *L* and *M* from those used in the 'world' calculus of the preceding section.) The Diodorean-modal thesis now to be proved for ending time is *CLMpMLp*, 'If it is and always will be that it is or will be that *p*, then it is or will be that it is and always will be that *p*'.[1] In endless time the antecedent of this could be true and the consequent false; this would be so if *p* were going to be true and false alternately for ever (it would then always be going to be true that *p*, but never going to be always true that *p*). But if time has an end, the antecedent will only be true if *p* is true at the end of time, and when this is so the consequent will be true also. In proving this implication from our axiom, note first that it expands by the Diodorean definitions to

CKApFpGApFpAKpGpFKpGp.

Ordinary propositional logic will equate this with

CApFpCGApFpCNKpGpFKpGp,

and in this, *NKpGp* = *ANpNGp* (de Morgan), i.e. *ANpFNp*, and a commutation turns the whole into

CApFpCANpFNpCGApFpFKpGp,

and this amounts (by *ECApqrKCprCqr*) to the conjunction of the following four formulae:

(1) *CpCNpCGApFpFKpGp*
(2) *CpCFNpCGApFpFKpGp*
(3) *CFpCNpCGApFpFKpGp*
(4) *CFpCFNpCGApFpFKpGp.*

[1] In D. C. Makinson's 'There are Infinitely many Diodorean Modal Functions', *Journal of Symbolic Logic*, vol. 31 (1966), pp. 406–8, this formula is mentioned as typical of a system D*, for the mirror-image of Diodorean modality in beginning time.

Of these, (1) holds trivially because of its contradictory antecedents, and in the rest we may use either an antecedent *Fp* or an antecedent *FNp* to obtain *FGKpNp* by *CFpFGq*, and proceed as follows

C	(1)	*FGKpNp*	
C	(2)	*GApFp*	
K	(3)	*FKApFpGKpNp*	(1, 2, *CKGpFqFKpq*)
K	(4)	*FAKpGKpNpKFpGKpNp*	(3, *CKApqrAKprKqr*)
K	(5)	*AFKpGKpNpFKFpGKpNp*	(4, *EFApqAFpFq*)
K	(6)	*AFKpGpFKFpGNp*	(5, *CKpqp*, *CKpqq*)
	(7)	*FKpGp*	(6, *NFKFpGNp*).

Turning now to the U-calculus version of the theory of ending time, the condition on *U* which we need to add is

(A) *CUabΣcKUacΠdNUcd*,

'If any instant is later than *a*, then some instant later than *a* has no instant following it'. If we were not leaving open the possibility of a branching future, we might have used the simpler *ΣaΠbNUab*, 'Some instant has no instant following it', but with branching time this would be consistent with some branches *not* ending. The longer formula (A) ensures that every branch there may be has an end. (To see this, suppose some branch *hasn't* an end. Then, starting from some point *a* on this branch, there will be a point *b* further along for which we have the antecedent *Uab*, but no point *c* on the branch which is without any points *d* beyond it, i.e. no point *c* satisfying the consequent *KUacΠdNUcd*.) If (A) is a law (and so implicitly preceded by *ΠΠb*), the shorter formula will follow from it. For

(B) *ΠbCUabΣcΠdNUcd*

follows from (A) in any case; we have by quantification theory *AΣbUabΠbNUab*; and *ΣcΠdNUcd* follows from (B) and *ΣbUab*, and from *ΠbNUab* immediately.

The corresponding tense-logical formula *CFpFGq*, supposed true at *a*, is equivalent by U1–U4 to

(C) *CΣbKUabTbpΣcKUacΠdCUcdNTdq*,

and this by quantification theory to

(D) *CUabCTbpΣcKUacΠdCUcdNTdq*,

which we prove from (A) thus:

C	(1)	*Uab*	
C	(2)	*Tbp*	
ΣcK	(3)	*Uac*	(1, (A))
K	(4)	*ΠdNUcd*	(1, (A))
	(5)	*ΠdCUcdNTdq*	(4, *CNpCpq*).

Conversely we obtain (A) from (D) by letting *p* and *q* in the latter be *Cpp*, *TbCpp* being detachable as a thesis, and *CUcdNTdCpp* being equivalent to *NUcd* (since it makes *Ucd* imply an absurdity).

Apart from the special axioms under consideration, the only tense-logical formulae used in all these proofs are ones provable in K_t.

It is of some interest to relate our formulae to McTaggart's characterization of beginning and ending time. What he says is that 'if the time-series has a first term, that term will never be future, and if it has a last term, that term will never be past'.[1] Concentrating on the second clause, what first strikes one about it is that ending time, as characterized by McTaggart, does not obey the tense-logical law that was later enunciated by J. N. Findlay,[2] that whatever is or has been or will be the case, sooner or later will have been the case, *CAApPpFpFPp*. The exception, in ending time, is anything that is true only at time's last instant; i.e. it is the last of the series of 'worlds'. If time is linear, then each instantaneous total world-state is something which is or has been or will be the case, i.e. we have *ΠaAAaPaFa*, but there is a world-state, the last one, which will never have been the case, *ΣaNFPa*. McTaggart's formula *ΣaNFPa*, i.e. *ΣaGNPa*, follows in our ending-time system by

$$(A).\ \rightarrow \Sigma a \Pi b N U a b \rightarrow \Sigma a \Pi b C U a b T b N P a$$

$$= \Sigma a T a G N P a = \Sigma a L C a G N P a \rightarrow \Sigma a L C P a P G N P a$$

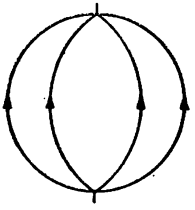
$$\rightarrow \Sigma a L C P a N P a \rightarrow \Sigma a L N P a \rightarrow \Sigma a G N P a.$$

3. *The east–west and north–south views of time.* One variation of the ending-time theory is worth mentioning in passing. In 1965 C. L. Hamblin described a version of *circular* time according to

[1] Op. cit. § 329 n.

[2] In 'Time: a Treatment of Some Puzzles', in A. G. N. Flew's *Logic and Language*, First Series (Blackwell, 1951), pp. 37–54.

which, if we go far enough into the future we shall find ourselves not in the future at all but in the past. He called this the 'East–West' conception of circular time, 'in the sense in which California is east but not west of Sydney, and west but not east of Manchester'. E. J. Lemmon then suggested that we might also have a 'North–South' conception of time, and this would be quite different, and not circular at all. The geometrical picture would be of this sort:



The North Pole would represent the end of time, and the South Pole would represent its beginning, and time would not be linear but branching, only the branches would all converge again. The usual formula for such convergence in the future is *CFGpGFp*, or in the U-calculus *CUabCUacΣdKUbdUcd*, and for the past *CPHpHPp* or *CUbaCUcaΣdKUdbUdc*, but in the present case these formulae will not do. Concentrating on the future, the convergence U-formula says that if *b* and *c* are both later than *a* then they have at least one later moment in common; but if time also ends, one of them might have no later moment at all. This combination does not result in a strict contradiction, but it entails that no moment is later (and so none earlier) than any other, i.e. that there is no future and no past, and this is not the general 'North–South' picture, though it is a limiting case of it. Technically, if we combine either *CUabCUacΣdKUbdUcd* with ending, or the reverse convergence formula with beginning, we can deduce a contradiction from *Uab*, and so prove *NUab* as a theorem. Doing it the first way, we have

C	(1)	*Uab*	
K	(2)	*ΠcCUacΣdKUbdUcd*	(1, convergence)
**K*	(3)	*ΣcKUacΠdNUcd*	(1, ending)
K	(4)	*ΠcCUacΣdUcd*	(2)
*	(5)	*NΣcKUacΠdNUcd*	(4).

Correspondingly, if we combine either *CFGpGFp* and *CFpFGq*

or *CPHpHPp* and *CPpPHq*, we can prove both *NFp* and *NPp* as theorems. Doing it the first way (using auxiliary theses from K_t),

1.	*CFGpFGNp*	(*CFpFGq*, *p*/*Gp*, *q*/*Np*)
2.	*CFGpNGFp*	(1, *FGN* = *NGNGN* = *NGF*)
3.	*NFGp*	(2, *CFGpGFp*, *CCpNqCCpqNp*)
*4.	*NFp*	(3, *CFpFGq*, *q*/*p*)
5.	*HFCpp*	(p.c., *CpHFp*)
6.	*CPpPKpFCpp*	(5, *CHpCPqPKqp*)
7.	*CPpPFCpp*	(6, *CPKpqPq*)
8.	*HNFCpp*	(4, *p*/*Cpp*; RH)
9.	*NPFCpp*	(8, *HN* = *NP*)
*10.	*NPp*	(7, 9).

We have here, in effect, that trivialization of tense logic which consists in interpreting *Gp* and *Hp* as the tautological function *Cpp*, and *Fp* and *Pp* as the contradictory function *NCpp*; i.e. K_t+*CFGpGFp*+*CFpFGq* is equivalent to the system obtained by adding these definitions to propositional calculus.

On the U-calculus side, the required amendment to the usual future-convergence formula is, I think,

(E) *CUabCUacCUbdCUceΣfKKUafUbfUcf*,

i.e. if *b* and *c* are both later than *a*, then *if b and c have any later moments at all*, all three of *a*, *b*, *c* have a common later moment. (If we assume *U* transitive, the *Uaf* in the consequent is not needed.) This holds in non-trivial as well as trivial forms of 'North–South' time. The equivalent tense-logical formula is *CFKpFqCFKrFsFKPpPr*. The U-calculus translation of this is

CΣbKUabKTbpΣcKUbcTcq
-CΣdKUadKTdrΣeKUdeTes
-ΣfKUafKΣgKUgfTgpΣhKUhfThr,

which we may contract to

(F) *CUabCTbpCUbcCTcqCUadCTdrCUdeCTes*
-ΣfKUafKΣgKUgfTgpΣhKUhfThr

We obtain this from (E) as follows:

$$
\begin{array}{llll}
C\,(1)\ Uab & C\,(2)\ Tbp & C\,(3)\ Ubc & C\,(4)\ Tcq \\
C\,(5)\ Uad & C\,(6)\ Tdr & C\,(7)\ Ude & C\,(8)\ Tes \\
\Sigma fK\,(10)\ Uaf & & & \\
K\,(11)\ Ubf & & (1, 5, 3, 7, (E)) & \\
K\,(12)\ Udf & & & \\
K\,(13)\ \Sigma gKUgfTgp & & (11, 2) & \\
(14)\ \Sigma hKUhfThr & & (12, 6). &
\end{array}
$$

And we obtain (E) from (F) by making the substitutions p/b, q/c, r/d, s/e and proceeding thus:

$$
\begin{array}{llll}
C\,(1)\ Uab & C\,(2)\ Tbb & C\,(3)\ Ubc & C\,(4)\ Tcc \\
C\,(5)\ Uad & C\,(6)\ Tdd & C\,(7)\ Ude & C\,(8)\ Tee \\
\Sigma fK\,(9)\ Uaf & & & \\
K\Sigma gK\,(10)\ Ugf & & (1\text{–}8,\ (F)) & \\
K\,(11)\ Tgb & & & \\
K\,(12)\ Igb & & (11, CTabIab) & \\
(13)\ Ubf & & (10, 12) & \\
\Sigma hK\,(14)\ Uhf & & & \\
K\,(15)\ Thd & & (1\text{–}8,\ (F)) & \\
K\,(16)\ Ihd & & (15, CTabIab) & \\
(17)\ Udf & & (14, 16), &
\end{array}
$$

then detaching the antecedents 2, 4, 6, 8 (i.e. *LCbb*, etc.) and dropping from the consequent all the conjuncts but 9, 13, and 17.

Summing up, 'North–South' time can perhaps be characterized by subjoining to K_t the axioms *CGpGGp* (for transitivity) with *CFKpFqCFKrFsFKPpPr* and *CFpFGq* and their images.

4. *Ending time and 'dead' time.* Von Wright has observed that the apparently more restricted axiom for ending time, *CFpFNFp*, looks as if it might not express that view, but rather the view that the truth is endlessly changing—that if anything is true, it is not true more than once. It might seem, in short, to be compatible with the idea that every particular thing will indeed cease to be true, but only by being replaced by some different particular thing. But *CFpFNFp* lays down this 'Everything will cease to be true' for *all* propositions, even logically true ones, and *these* surely could only cease to be true by time itself ending

(their ceasing to be true would *be* time's ending). If the *p*'s in the formula were confined to 'atomic' propositions, it might be susceptible of von Wright's interpretation; we would have something like it, too, if we replaced the *p*'s by *a*'s. For logically true propositions are not substitutable for world-variables, and *CFaFNFa* would indeed assert, not that there will be a last world-state, but that no future world-state will be repeated.

We come closer to ending time if we make an assumption that is in a way the opposite of von Wright's, namely that we will eventually reach a state which will never cease to be the world's total state, i.e. sooner or later time will 'stand still', in the sense that no further change will occur in anything whatever. Some philosophers have spoken as if there would be no more than a verbal difference between this state of affairs and time's coming to an end, and some have denied this. For example, McTaggart would apparently equate the two, since he says flatly that 'there could be no time if nothing changed'.[1] Locke, on the other hand, sharply distinguishes the regular motions which we use for *measuring* duration from the duration itself which is being measured, and insists that 'having from the revolutions of the sun got the ideas of certain lengths of duration, we can in our thoughts add such lengths of duration to one another as often as we please', and 'this we can continue to do on, without bounds or limits, and proceed *in infinitum*, and apply thus the length of the annual motion of the sun to duration, supposed before the sun's *or any other motion* had its being'[2] (last italics mine). And, presumably, after.

Formally, a short decision between these two positions would be that an end of change *can* be distinguished from an end of time. The U-calculus formula expressing the view that (on any branch that we may be on, if time has branches) we shall sooner or later reach an end of *change*, would be

(G) $CUab\Sigma cKUac\Pi dCUcdIcd$,

which we may compare with

(A) $CUab\Sigma cKUac\Pi dNUcd$,

the formula for ending *time*. (G) follows trivially from (A), but not vice versa. The tense-logical counterpart of (G) is the thesis

[1] Op. cit., § 309.

[2] *Essay concerning Human Understanding*, Book II, ch. xiv, §§ 23, 27.

CFpFCqGq ('If anything will be the case, then it will sooner or later be the case that whatever is true will be so for ever after'). Preceded by *Ta*, this expands to

$$C\Sigma bKUab\,Tbp\Sigma cKUac\,CTcq\Pi dCUcd\,Tdq,$$

which is deductively equivalent to

(H) $CUabCTbp\Sigma cKUac\Pi dCUcdCTcq\,Tdq.$

This may be proved from (G) as follows:

C	(1)	*Uab*	
C	(2)	*Tbp*	
ΣcK	(3)	*Uac*	(from (1) by (E)).
K	(4)	*ΠdCUcdIcd*	
	(5)	*ΠdCUcdCTcqTdq*	(from(4)).

Conversely, if in (H) we put *b* for *p* and *c* for *q*, we may detach *Tbb* and *Tcc* to obtain

$$CUab\Sigma cKUac\Pi dCUcd\,Tdc.$$

This yields (G) by *CTdcIcd*. And *CFpFCqGq* is related to the ending-time tense-logical formula *CFpFGq* as (G) is to (A), i.e. it follows from it, but not vice versa. (If we let $Fp = Gp = Pp = Hp = p$, all of K_t, and *CFpFCqGq*, become propositional calculus tautologies, but not *CFpFGq*.)

So far we seem to be with Locke and against McTaggart. It might nevertheless seem that the procedure adopted in our first section, of simply identifying an instant with the totality of what would be said to be the case 'at' that instant, immediately commits us to McTaggart's view that the end of change is only verbally different from the end of time. But the matter is not quite as simple as that. Certainly, given the way we use 'instant', we are committed to the view that, once an instant of 'dead' time is reached, there will be no *other* instant after this one; i.e. that there can only be one instant of 'dead' time. But, given the way we use 'instant', this means no more than that, once 'dead' time is reached, nothing will be true but what is true already. The question as to whether anything at all 'will' be true is still an open one, and one which, since 'it will be the case that' is not defined in terms of 'instants', it is perfectly possible to

formulate, and to answer either way, without referring to 'instants' either in the above sense or in some more Platonistic one.

We could, indeed, immediately dismiss the idea of 'dead' but not ending time if we assumed that no instant is later than itself. This is intuitively obvious (given the above conception of an instant), and formally, if we make *NUaa* an axiom, we can deduce the ending-time formula (A) from the dying-time formula (G) as follows:

	C (1)	*Uab*	
ΣcK	(2)	*Uac*	
K	(3)	*ΠdCUcdIcd*	(1, (G))
K	(4)	*ΠdCUcdUdd*	(3)
	(5)	*ΠdNUcd*	(4, *NUdd*).

But to say that no instant, in *this* sense of 'instant', is ever in its own future, is plainly to beg the question of whether dead but not ending time is possible, since it is just to say that no world-state is ever repeated. There are in fact consistent tense logics (ones for circular time) in which it is laid down that *every* instant is in its own future.

5. *Discreteness, denseness, and continuity.* There are, all the same, definite limits to our freedom to combine dying with non-ending time. In particular, too extreme an absolutism will be self-defeating. Locke, for example, seems to have held not only that there is a difference between time's going on but nothing happening, and time's not going on at all, but also that even in 'dead' time there is a difference between intervals in respect of their length. But if this is so, the instants of this supposedly 'dead' time, coming after a 'live' time, will not be identical in the above sense, since one thing that will be the case at any instant *a* of 'dead' time will be that the state of the world was something other than *a* such-and-such an interval ago, and these intervals will be different for each *a*, so that the *a*'s will *not* all of them be quite the same proposition: indeed, no one of them will be quite the same as any other.

We have a special case of this when time is supposed to be discrete, since this gives us built-in units of temporal distance. In discrete time we just cannot without contradiction suppose

an endless succession of identical world-states to follow after a state which is different from any of them; indeeed, we cannot even get two. For one thing that will be true in the first state in this supposed succession, which we may suppose to be the state *a*, is that the immediately preceding state, whatever it was, was not *a*; but in the next state in the supposed succession, one thing that will be true is that the immediately preceding state *was a*; so these two states are not after all identical, and the supposition that they are is impossible. To formalize this reasoning, we may make use of a tense logic due to Dana Scott, in which the usual tensed forms are supplemented by Tp for 'It will be the case at the next instant that *p*', and Yp for 'It was the case at the instant just gone that *p*' and in which we have (among others) the following laws:

TG: $CGCpqCTpTq$
TY: $CTYpp$.

(I select these two because in themselves they do not commit us either to time's infinity or its denial. Scott's complete system in fact contains the laws $CGpTp$ and $CHpYp$, which do commit us to time's infinity, but I shall not make use of these in what follows.) Given this much, we have the following proof, in which the antecedents express the assumptions that *a* is the case now, that it will be the case at the next instant, and that it was not the case at the instant just past:

C	(1)	*a*	
C	(2)	Ta	
C	(3)	YNa	
K	(4)	$LCaYNa$	(1, 3, *CKapLCap*)
K	(5)	$GCaYNa$	(4, *CLpGp*)
K	(6)	$CTaTYNa$	(5, TG)
K	(7)	$TYNa$	(6, 2)
	(1)	Na	(7, TY),

in which the conclusion contradicts the hypothesis.

If time is *not* discrete, then if 'live' time is followed by 'dead' time, either (i) there is a first moment of 'dead' time but no last moment of 'live' time, or (ii) there is a last moment of 'live' time but no first or 'dead' time, or (iii) there is neither a last moment of 'live' time nor a first of 'dead' (the third alternative is only

possible if time is dense but not strictly continuous). (i) can be easily excluded, since at the first moment of 'dead' time it will be the case that all previous moments were 'live', but at subsequent moments this will not be so, contradicting the definition of 'dead' time as time throughout which whatever is true at any moment is true at any moment future to that one. Or, more accurately, it entails that there can be *no* moment of 'dead' time which is future to the first (if there were the time would not be 'dead'), so that this kind of 'death' *is* identical with time's having an end. Formally the proof is:

C	(1)	*a*	
C	(2)	*Ga*	
C	(3)	*NPa*	
K	(4)	*LCaNPa*	(1, 3, *CKapLCap*)
K	(5)	*GCaNPa*	(4, *CLpGp*)
K	(6)	*GPa*	(1, *CpGPp*)
K	(7)	*GNa*	(5, 6, *CGCpNqCGqGNp*)
K	(8)	*GKaNa*	(2, 7)
K	(9)	*GNp*	(8, *CKpNpq*, RGC)
	(10)	*NFp*	(9, *GN* = *NF*),

where *p* is any arbitrarily chosen proposition whatever.

It may be thought that in this last proof the antecedent (3) is stronger than our supposition warrants—we are only supposing that the time immediately leading up to the first 'dead' instant *a* is one throughout which *a* is false; we don't have to suppose *a* never to have been true before. But it can be shown that if we suppose *a* to have been true previously, with an intervening interval of falsehood, it will be inconsistent to suppose that *a* will be always true henceforward (where *a* is a world-state proposition). For we have

C	(1)	*a*	
C	(2)	*PKNaPa*	
K	(3)	*PKHFNaPa*	(2, *CpHFp*)
K	(4)	*PPKFNaa*	(3, *CKHpPqPKpq*)
K	(5)	*PPLCaFNa*	(4, *CKpaLCap*)
K	(6)	*LCaFNa*	(5, *CPLpLp* twice)
K	(7)	*FNa*	(6, 1)
	(8)	*NGa*	(7, *FN*= *NG*).

Informally, if *a* completely characterized some previous world-state, part of its content would be that it was going to be false, so that if *this very same a* completely characterizes the present world-state, it cannot be that it will never be false again, i.e. that there will be no further alterations in the world.

If, however, we make supposition (ii) or (iii), above, i.e. if we suppose 'dead' time to be preceded by 'live' time but in such a way that between any moments of 'dead' and 'live' time there is an earlier moment of 'dead' time, the above proof of time's ending (two paragraphs back) cannot be constructed. Possibly, under these circumstances, some other line of proof will turn out to be available, but on the whole it looks as if we *can* distinguish between time's going 'dead' and time's coming to an end, provided that we do not (like Locke) suppose time to have an intrinsic 'metric' (in which case the supposition of time's going *completely* 'dead' is self-contradictory), and provided that we do not suppose there is a *first* moment of 'dead' time. If, finally, we redefine 'dead' time as time throughout which there is no change in the truth-value of those propositions which have no past or future-tense operators in them (a suggestion made to me by Dr. P. Nidditch), there would seem to be no *logical* incoherence in supposing 'live' time to be followed by an indefinitely long period of 'dead' time even *with* a first moment. For the negative results above depend entirely on the truths at a given instant including past-tense and future-tense ones.

XI

TENSE LOGIC AND THE LOGIC OF EARLIER AND LATER

W. V. QUINE once wrote a paper entitled 'Three Grades of Modal Involvement'.[1] Tense logic, like modal logic, is something about which some philosophers have misgivings, and like modal logic, it may be presented in a more or a less accommodating manner—it has, in Quine's terminology, its own 'grades of involvement'. I want to present here four such 'grades of tense-logical involvement', by presenting a series of calculi involving the notion of being true (or as Rescher says, 'realized') at an instant, making more and more controversial assumptions at each main stage.

1. *Tense logics and U-calculi.* We may begin from the juxtaposition of two calculi of E. J. Lemmon's, one a 'minimal' tense logic and one a minimal calculus of the earlier-later relation. Sometimes[2] the tense logic, called K_t, is axiomatized with *F* (for 'It will be the case that—') and *P* (for 'It has been the case that—') as primitive symbols, with *G* (for 'It will always be the case that—') defined as *NFN*, and *H* (for 'It has always been the case that—') as *NPN*. Here I take *G* and *H* as primitive, and define *F* as *NGN* and *P* as *NHN*, the postulates then becoming

RG: $\vdash\alpha \rightarrow \vdash G\alpha$	RH: $\vdash\alpha \rightarrow \vdash H\alpha$
1.1. *CGCpqCGpGq*	1.2. *CHCpqCHpHq*
2.1. *CNGNHpp* (*CFHpp*)	2.2. *CNHNGpp* (*CPGpp*),

these being subjoined to propositional calculus with substitution and detachment. In the associated earlier-later calculus, using *Tap* for 'It is the case at the instant *a* that *p*' and *Uab* for 'The

[1] Now printed in his *The Ways of Paradox* (1966), pp. 156–74.
[2] As in Papers IX and XIII.

instant *a* is earlier than the instant *b*', we subjoin to propositional calculus and quantification theory these postulates for *T*:

T1. *CTaCpqCTapTaq*

T2.1. *CTaNpNTap* T2.2. *CNTapTaNp*

and these for *U*:

UT1. *ETaGpΠbCUabTbp*

UT2. *ETaHpΠbCUbaTbp.*

In the T-calculus we may prove the converse of 1, so that 1 and the 2's may be replaced by a pair of equivalences, and in the whole we may prove the formulae *ETaFpΣbKUabTbp* and *ETaPpΣbKUbaTbp*, relating *U* to *P* and *F*, and giving us the postulates of the U-calculus appropriate to a system with these as primitive. Whether we use the one basis or the other, the tense-logical theses which are provable, preceded by *Ta*, in the minimal U-calculus (e.g. *TaCFHpp*) are precisely those of the minimal tense logic K_t, and the addition of various special conditions on *U* (e.g. *ΠaΣbUab*, giving every instant a *U*-successor) makes possible the proof of new theses of tense logic, preceded by *Ta*.

Here the elementary forms *Tap* and *Uab* of the earlier-later calculus are what Rescher[1] calls 'chronologically definite' propositions, whose truth-value is independent of time; so of course are truth-functions and quantifications of these. If we regard these as propositions *par excellence*, the tensed formulae for which the variables *p*, *q*, *r*, etc. may stand may be regarded as *predicates* of the instants 'at' which they are (perhaps loosely) said to be true. Tense logic, we might say, is a logic of pure predicates which are artificially torn away from their subjects and given a spurious independence. Its theses only make sense if we understand them to be implicitly preceded by a *Ta*, as they are explicitly in the U-calculus.

With this conception of the relation between the two calculi, we ought strictly speaking to use different symbols for the *C*'s and *N*'s which occur inside and outside the *Ta*'s; e.g. T1 ought properly to be written as *𝒞TaCpq𝒞TapTaq*, T2.1 as *𝒞TaNp𝒩Tap*, and UT1 as *𝒞TaGpΠb𝒞UabTbp*, with the script

[1] N. Rescher, 'The Logic of Chronological Propositions', *Mind*, vol. 75, no. 297 (January 1966), pp. 75–96.

letters for proposition-formers and the italics for predicate-formers. Indeed, the four equivalences consisting of T1 and its converse, the T2's, and the UT's, may be replaced by *definitions* of the predicate-formers in terms of the proposition-formers, e.g. the T2's by

$$TaNp = \mathcal{N}Tap,$$

equating 'The instant a is (non-p)-ish' with 'It is not the case that the instant a is p-ish'; and UT1 by

$$TaGp = \Pi b\mathscr{C}UabTbp,$$

equating 'The instant a is (p-for-evermore)-ish' with 'The instant a is earlier than none but p-ish instants'. T here expresses no more than the attachment of a predicate to its subject, and could be replaced by juxtaposition and bracketing, as in this further transformation of the T2's:

$$(\mathcal{N}p)a = \mathcal{N}(pa).$$

The expansion of pa, 'a is p-ish', to Tap, 'It is true at a that p', is only a special case of the expansion we sometimes make of 'x ϕ's' to 'It is true *of* x that it ϕ's'; 'at', we might say, is just this 'of'.

This is what I call the first or lowest grade of tense-logical involvement. Philosophers who are uneasy about tense logic will almost certainly find little in this amount of it to worry about. And there is a nice economy about it; it reduces the minimal tense logic to a by-product of the introduction of four definitions into an ordinary first-order theory, and richer systems to by-products of conditions imposed on a relation in that theory.

It is not *quite* to be taken for granted, however, that the U-calculus is philosophically simpler than a more substantial tense logic would be. For one thing, the U-calculus has two sorts of variables where tensed propositional logic has only one. And if it be replied that the extra ones are just name-variables, which will have to be introduced sooner or later anyway, the answer to that is that they are names of a very odd kind of entity. Some of us at least would prefer to see 'instants', and the 'time-series' which they are supposed to constitute, as mere logical constructions out of tensed facts. With this motivation, let us try going a little further.

2. *Assignments-to-instants as omnitemporal tensed propositions.* We may begin by ignoring the form *Uab* (we shall of course come back to it), and concentrating on the relations between the form *Tap*, 'It is the case at the instant *a* that *p*', and the simple *p*. If we treat both of these as genuinely propositional, the form *Tap* will be simply a *special case* of the sort of thing for which the variable *p* can stand, and it will make sense to substitute such formulae for the *p*'s and *q*'s both of tense logic and of the U-calculus. *TbTap*, for example, will be as well formed as the simple *Tap*, though there will be a certain vacuity about the initial *Tb*, since we may presume that if it is true at any time, e.g. now, that *p* is true at the instant *a*, this will be equally true at any other time. What this amounts to is that instead of ruling out such forms as *TbTap*, we explicitly lay it down that *TbTap* is equivalent to the plain *Tap*. Again, if forms like *Tap* and *ΠbTbp* are on the same level as the simple *p*, they may occur as arguments of a single truth-function, as in, for example, *KTapq*, *CΠaTapp*.

Allowing ourselves these liberties (which represent the second grade of tense-logical involvement) we may add to T1 and the T2's the following futher postulates for T:

RT: $\vdash\alpha \rightarrow \vdash T a\alpha$
T3. *CΠaTapp*
T4. *CΠaTapTbΠaTap*
T5. *CTapTbTap.*

What we now have is equivalent to the System SI of Rescher. In the T-calculus thus enriched, it is easy to prove the converses of T4 and T5; and if we introduce the form $L\alpha$ as an abridgement of $\Pi a T a\alpha$ in those cases where *a* does not occur free in α, we may prove for this *L* the following:

RL: $\vdash\alpha \rightarrow \vdash L\alpha$
L1. *CLCpqCLpLq* (from T1 by quantification theory and Df. *L*)
L2. *CLpp* (from T3 by Df. *L*)
L3. *CNLpLNLp*,

the last a little tortuously (using T5). These are Gödel's postulates for the Lewis system S5, which we therefore have for this *L*.

If we now introduce the form *Uab* and draw upon UT1 and UT2, we may prove, for the above *L*, the theses

L4. *CLpGp*
L5. *CLpHp*.

For we obviously have *CΠbTbpΠbCUabTbp* and *CΠbTbpΠb CUbaTbp*, and so *CLpTaGp* and *CLpTaHp* by UT1 and 2; from these we have *CLpΠaTaGp* and *CLpΠaTaHp* by *Π2a*, and finally L4 and L5 by T3.

We may also, drawing upon UT1 and 2, prove certain results about the prefixing of tense-operators to the form *Tap*. It might be thought that, since *Tap* is 'omnitemporally' true, the forms *FTap*, *PTap*, *GTap*, and *HTap* should all be equivalent to the plain *Tap*, but it is not quite as simple as that. The key point is that UT1 and UT2 leave it quite open whether time has or has not a beginning or an end. If there *were* an end of time, *FTap* would be false at the end of time regardless of whether *Tap* itself were true or false, since anything at all beginning with 'It will be the case that—' would be false then. In other words, *CTapFTap* might be false because although *Tap is* true, nothing whatever *will* be true, because there is now no future. And since *Gp* is equivalent to *NFNp*, *GTap* (= *NFTaNp*) would be vacuously *true* at the end of time regardless of the truth or falsity of *Tap*. This means that we have *CFTapTap* and *CTapGTap* but not their converses, and similarly with *P* and *H*. It has already been observed that from T1, the T2's, and UT1 and 2 we have *ETaFpΣbKUabTbp*; hence we have

C	(1)	*FTap*	
K	(2)	*ΣbKUabTbTap*	(1)
K	(3)	*ΣbKUabTap*	(2, Conv. T5)
K	(4)	*KΣbUabTap*	(3)
	(5)	*Tap*	(4).

And for the other, we clearly have *CTapΠbCUabTap*, and so, by T5, *CTapΠbCUabTbTap*, i.e. *CTapGTap*. We may prove the converses, as we would expect, if we add the axiom *ΣbUab*, expressing time's forward infinity, and the converses of the corresponding theses in *P* and *H* if we add *ΣbUba*, expressing the infinity of the past.

To prove similar theorems about the form *Uab*, i.e. to prove *CFUabUab*, *CUabGUab*, and their images, it is necessary to add to UT1 and 2 the further postulate (analogous to T5)

UT3. *CUabTcUab*.

The converse of this (like that of T5) is provable.

Finally, if we enrich the U-calculus in the ways suggested, any tense-logical formula which we can prove, preceded by *Ta*, in the U-calculus, we can now prove without this prefix, by passing from *Taϕ* by U.G. to *ΠaTaϕ* and from this to *ϕ* by T3. So we no longer have merely *parallel* tense logics and U-calculi; the tense logics now appear as *parts* of the U-calculi, and this may prepare the way for treating the U-calculi as parts of the tense logics.

3. *Alternative interpretation of the enlarged system.* The characteristic feature of the present enlargement of the T-calculus, the placing of *Tap* and the plain *p* in the same syntactical category, may well be felt by some to be *the* step which must *not* be taken. Certainly it is essential to what I want to do next. But in itself it does not amount to much, and a person determined to treat tensed propositions as predicates could still do something with the present calculus.

To see what such a person might do with it, we should first observe that even from the point of view of such a person, the calculus of Section 1 has a certain insufficiency about it. It does not suffice to prove, for example, T3 preceded by *Ta*, i.e. *TaCΠaTapp*; but surely even a person who regarded tensed *p*'s as predicates of instants would want to say that it is true of any arbitrary instant *a* that it is '*p*-ish if every instant is *p*-ish'. But the calculus of Section 1 does not even provide for the *formation* of predicates of this sort (i.e. ones like '*p*-ish if every instant is *p*-ish'). In ordinary predicate calculus we may form complex predicates not only by truth-functionally combining simpler ones (as in the predicate 'stands and smokes'), but also by truth-functionally combining predicates and propositions, e.g. we might predicate of John that he 'is coming if and only if Mary is coming'. The predicate-of-instants '— is *p*-ish if every instant is *p*-ish', mentioned a few lines higher up, is clearly of this sort; another of the same would be '— is *p*-ish if the instant *b* is *q*-ish'.

In defining predicate-formers in terms of proposition-formers we ought therefore to have not only

$$(Cpq)a = \mathscr{C}(pa)(qa)$$

but also

$$(Cp\alpha)a = \mathscr{C}(pa)\alpha$$

and

$$(C\alpha p)a = \mathscr{C}\alpha(pa),$$

where the schematic letter α represents any 'genuine' (i.e. chronologically definite) proposition. Or, doing it with a T-calculus, we could add to the axiom T1 the axiom-schemata

TS1. $\vdash ETaCp\alpha CTap\alpha$

TS2. $\vdash ETaC\alpha pC\alpha Tap.$

We could then obtain, for example, the above-mentioned thesis $TaC\Pi aTap$ by TS2 from $C\Pi aTapTap$, which we have from quantification theory. And if, as is done in some systems of propositional calculus, we defined 'Not p' in terms of a constant false proposition 0, the equivalence of $TaNp$ and $NTap$ asserted by our T2's would become simply that case of the schema TS1 in which α is 0.

But if someone wanted to replace the *schemata* TS1 and 2 by *axioms* for the key types of α that the system contains, he might well calculate that it would be less cumbersome to do it by running p's and α's together and preventing this confusion from doing any harm by the 'evacuating' postulates T4 and T5 (and UT3). This would mean regarding the theses of the enlarged calculus as all of them preceded by an unexpressed Ta (or ΠaTa) attaching to an arbitrary instant any ostensibly unattached 'predicates' p, q, r, etc. that a thesis might contain, the attachment to it also of 'genuine' propositions being a convenient but empty formality (rather like vacuous quantification).

4. *Instants as propositions.* That the last step forward is not a very sensational one, is sufficiently indicated by the fact that we are still left with variables ostensibly representing named or nameable instants as well as ones representing propositions. What I shall call the third grade of tense-logical involvement consists in

treating the instant-variables *a*, *b*, *c*, etc. as also representing propositions. We might, for example, equate the instant *a* with a conjunction of all those propositions which would ordinarily be said to be true at that instant, or we might equate it with some proposition which would ordinarily be said to be true at that instant only, and so could serve as an index of it. When we do this we need of course to find some suitable interpretation for the form *Tap*, as it scarcely makes sense to speak of one proposition as being true 'at' or 'in' another proposition. We might say, for example, that it is true 'at' or 'in' *a* that *p*, if the proposition *a* at all times implies that *p*; it can in fact be proved that *Tap* is logically equivalent to *LCap*. (This follows from the postulates already given, plus two that will shortly be added to them.)

This sounds a highly artificial procedure, but remember that what lies behind it is the belief that 'instants' are artificial entities anyhow, i.e. that all talk which appears to be about them, and about the 'time-series' which thay are supposed to constitute, is just disguised talk about what is and has been and will be the case. Certainly this revision of the concept of an instant turns quantification over instants into quantification over a certain sort of propositional variable, and it is important to notice that our third grade of tense-logical involvement commits us to this; but, while I shall not argue the pros and cons here, it seems to me, for reasons that have nothing specially to do with tense logic, that we have to admit such quantifications anyhow.

We take the instant-variables *a*, *b*, *c*, then, as propositional variables, substitutable in theses for the more general propositional variables *p*, *q*, *r*, etc.; though the converse substitution is not possible, since *a*, *b*, *c*, etc. stand only for those propositions which satisfy the postulates RT and T1–T5, together with two that are formulable now that instants are treated as propositions, namely

T6. *Taa*,

i.e. one of the propositions that is true 'at' the instant *a* is always the instant-proposition *a* itself; and

T7. *CTapCap*.

Of the theorems which are now provable, the most important are

T8. Ma ($= NLNa$)
T9. Σaa
T10. $CLCapTap$
T11. $CTapLCap$.

T8 may be proved as follows:

1.	$C\Pi bNTbaNTaa$	(U.I)
2.	$N\Pi bNTba$	(1, T6)
3.	$C\Pi bTbNa\Pi bNTba$	(T2. 1, $\Pi 1b$, $\Pi 2b$)
4.	$N\Pi bTbNa$	(3, 2)
5.	$NLNa$	(4, Df.L);

T9 as follows:

1.	$Ca\Sigma aa$	(quantification theory)
2.	$CTaaTa\Sigma aa$	(1, RT, T1)
3.	$Ta\Sigma aa$	(2, T6)
4.	$\Pi aTa\Sigma aa$	(3, UG)
5.	Σaa	(4, T3);

T10 thus:

C	(1)	$LCap$	
K	(2)	$\Pi bTbCap$	(1, Df.L)
K	(3)	$TaCap$	(2, U.I.)
K	(4)	$CTaNpTaKNpCap$	(3, $CTapCTaqTaKpq$)
K	(5)	$CTaNpTaNa$	(4; $CKNpCapNa$, RT, T1)
K	(6)	$NTaNa$	(T2.1, T6)
K	(7)	$NTaNp$	(5, 6)
	(8)	Tap	(7, T2.2);

and T11 thus:

1.	$CTbTapTbCap$	(T7, RT, T1)
2.	$CTapTbCap$	(1, T5)
3.	$CTap\Pi bTbCap$	(2, $\Pi 2b$)
4.	$CTapLCap$	(3, Df.L).

The addition of T6 and T7 also makes possible certain simplifications of the total system. For example, T3 can now be

replaced by the shorter T9 (Σaa), since T3 is provable from T9 and T7 as follows:

1. $C\Pi aTapCap$ (T7, $\Pi 1a$)
2. $CaC\Pi aTapp$ (1, p.c.)
3. $C\Sigma aaC\Pi aTapp$ (2, $\Sigma 1a$)
4. $C\Pi aTapp$ (3, T9).

We may also drop T4, though the proof of this is a little indirect. From S5 and quantification theory we can prove

$$C\Pi aLCapLCb\Pi aLCap,$$

which is what T4 may be transformed into in virtue of the equivalence of Tap and $LCap$ proved in T10 and T11. And T4 is not itself used in the proof of S5 for L or of T10 and T11.

We may also prove $CTabTba$ and $CTabCTbcTac$, which with T6 (Taa) show that when both its arguments are instant-propositions T behaves as an equivalence functor. Indeed we can prove (by T7 and induction on possible contexts of a and b) the metatheorem that $CTabCf(a)f(b)$ holds for any function $f(a)$ constructible in the system, so that Tab can be used to express a certain sort of identity between instants (Tab means that precisely the same propositions are true at a as at b). We might, indeed, have introduced a propositional identity-function Ipq with the usual postulates (Ipp, and $CIpqCf(p)f(q)$ for all functions of the system), and laid down $CTabIab$ instead of T7, thereby equating T (with instant-arguments) with this I (since T6, Taa, is clearly equivalent by identity-theory to $CIabTab$).

When we add the form Uab with its axioms UT1 and UT2, we can extend the above metatheorem $CTabf(a)f(b)$ to the new functions that we now have, either by introducing I and equating Tab with Iab, or (if we keep the original T7) by adding the further axiom

UT4. $CTabCUcaUcb$.

$CTabCUacUbc$ then becomes provable (making possible the desired extension of the metatheorem). So does UT3, which we can therefore drop as an axiom. We can also, in this extended UT-calculus, prove the theses

T12. $CUabTaFb$,
T13. $CTaFbUab$.

T12 as follows:

C (1) *Uab*
K (2) *Tbb* (T6)
K (3) *ΣcKUacTcb* (1, 2, E.G.)
(4) *TaFb* (3, UT1).

T13 equates by UT1 to *CΣcKUacTcbUab*, in which quantification theory equates the initial *CΣc* to *ΠcC*, and we have *ΠcCKUacTcbUab* by UT4.

T12 and T13 mean that we could *define Uab* as *TaFb*, and if we do so we can replace UT1–4 by the last section's L4 (*CLpGp*) and L5 (*CLpHp*) and the tense logic K_t. Proofs of the UT's from these and a system which will shortly be shown equivalent to RT and T1–7, may be found in my *Past, Present, and Future*. Moreover, when further tense-logical postulates are added to K_t, we can prove those conditions on *U* which make it possible to prove the added tense-logical postulates in the U-calculus; i.e. the traffic between tense logics and U-calculi, given the new T-calculus, is two-way. And since 'at' can no longer mean the predicational 'of', UT1 and UT2 cannot be replaced by definitions, and the movement from them to the tense logic K_t is less straightforward and natural than the converse one. U-calculi, in short, are now best thought of as by-products of the interaction of the T-calculus and tense logics.

We have still to derive these, however, from tense logic alone. As a step in that direction, but one which in itself commits us to no further 'tense-logical involvements', we shall develop the present T-calculus in a slightly different form.

5. *Reformulation with L primitive.* In view of T10 and T11, it is possible to develop the calculus sketched in the last section with *L* instead of *T* as the one non-tense-logical primitive, defining *Tap* as *LCap*. This has in fact already been done in *Past, Present, and Future*, using as postulates RL, L1–3, T8 (*Ma*), T9 (*Σaa*), and T2.2 (= *CNLCapLCaNp* = *ALCapLCaNp*). It is from these postulates, with K_t, L4, L5, and the definition of *Uab* as *TaFb* (= *LCaFb*), that UT1 and UT2 are there derived; it remains to show their equivalence to those of the last section. The *L* postulates were proved there and in the preceding section from the T ones; and as to the converse proofs,

(i) given that $Tap = LCap$, it is easy to prove RT, T1 and T4–7 in S5;

(ii) T2.1 follows from modal logic and T8(Ma);

(iii) T3 may be proved from T9 (Σaa) as it was in the T-calculus (since we have the crucial thesis T7 by (i)); and

(iv) of the implications $CL\mathrm{p}\Pi aLCap$ and $C\Pi aLCapL\mathrm{p}$, jointly equivalent to the definition of L in the T-system as ΠaTa, the first follows from modal logic and the second we prove thus:

1. $C\Pi aLCapp$ (T3)
2. $CL\Pi aLCapL\mathrm{p}$ (1, RL, L2)
3. $C\Pi aLLCapL\mathrm{p}$ (2)
4. $C\Pi aLCapL\mathrm{p}$ (3),

where 3 follows from 2 by the 'Barcan formula' $C\Pi aLf(a)L\Pi af(a)$, known to be provable from S5 and quantification theory, and 4 from 3 by $CL\mathrm{p}LL\mathrm{p}$, also known to be provable in S5. And if we define Iab as $LEab$, we may prove $CTabIab$ from S5, T2.2 and T8, or (defining Uab as $TaFb$, i.e. $LCaFb$) UT4 from S5, L4, and K_t.

The *consistency* of the whole system may easily be shown by interpreting $L\mathrm{p}$ as p, and the special variables a, b, c, etc. as standing for tautologies. The postulates then all become theses of the propositional calculus.

This L system is stratified as well as the T system, but on a different principle. The T system divides its postulates into those (T1, T2.1, T2.2) which do not even assume that Tap is out of the same syntactical box as the plain p (and if U is added, UT1 and UT2 do not assume this either); those (RT, T3–5, UT3) which do assume this for Tap but not for the plain a; and those (T6, T7—and when we add U, UT4) which assume it even for a. The postulates of the L system, on the other hand, divide into those (RL, L1–3 and L4, L5, and K_t when we add them) which do not require the special variables a, b, c, etc. for their formulation, and which thus apply to propositions generally; and those (T8, T9, T2.2) which do require these variables, and give in effect the conditions a proposition must meet to be substitutable for one of the special variables. It must be of such a sort that (i) any such proposition is at some time true (T8), (ii) at any given time some such proposition is true (T9), and (iii)

such a proposition permanently implies either the truth or the falsehood of any given proposition (T2.2).

With a little modification of the *L*-system we can do without special variables in favour of a function Qp defined as $KMp\Pi qALCpqLCpNq$; i.e. Qp asserts that p satisfies T8 and T2.2. We then drop these two postulates, and replace all formulae of the form $\Sigma af(a)$ by $\Sigma pKQpf(p)$, and all of the form $\Pi af(a)$ by $\Pi pCQpf(p)$, and in theses with free special variables we replace these by ordinary propositional variables and conditions of the form CQp at the beginning of the thesis. For example, T2.1, $CLCapNCLaNp$ becomes $CQqCLCqpNLCqNp$. But we still need the postulate T9, in the form $\Sigma pKQpp$. This can be very slightly shortened by dropping from Qp, when expanded by its definition, the conjunct Mp, which follows from the new conjunct p. A still shorter equivalent of $KQpp$ which may be used in this axiom is $Kp\Pi qCqLCpq$. The axiom as thus amended asserts that there is something which is now the case, and which is at no time true without everything that is now the case being then true also. (This would fit, for example, a conjunction of all the propositions that are now true; it would also fit a proposition which is true now but not at any other time; cf. our original account of 'instant-propositions' at the beginning of Section 4.) This modification of the L-calculus, however, we can ignore in what follows.

6. *The tense-logical definition of L.* We would reach a fourth grade of tense-logical involvement if we could give a tense-logical definition of *L*. As to this, it has been shown in *Past, Present, and Future* that (i) if we are prepared to use a richer tense logic than K_t, quite simple tense-logical definitions of *L* are possible, e.g. if we assume that time is like a single non-branching line and that the earlier-later relation is transitive, we may define Lp, 'It is the case at all times that p', as $KKpHpGp$, 'It is and always has been and always will be the case that p'; and that (ii) even with the non-committal system K_t, if we enlarge our symbolic apparatus a little we can give a tense-logical definition of *L*, provided that we do not allow that there may be several distinct and independent time-series (in which case there would be 'times' which we could not locate from 'now' by any combination of 'will bes' and 'has beens'). But, under (ii), it is

possible to improve a little upon the definition given in *Past, Present, and Future*, and upon the proofs there given of the postulates RL and L1–5.

We first define the form $L^n p$ inductively as follows:

$$L^0 p = p$$
$$L^{n+1} p = KHL^n pGL^n p.^1$$

We then introduce quantifiers binding the n's, and obeying the usual rules, and define Lp as $\Pi n L^n p$. This is equivalent to defining $M^n p$ inductively by

$$M^0 p = p$$
$$M^{n+1} p = APM^n pFM^n p$$

and defining Mp as $\Sigma n M^n p$, i.e. we say that it is true at *some* time that p if and only if either p is itself true now or it is true preceded by *some* combination of 'will be's and 'has been's.

$L^1 p$, it may be observed, is $KHpGp$, and it is easy to show that for this functor, given K_t for H and G, we may prove all but L2 ($CLpp$) of the postulates of the 'Brouwersche' modal system, i.e. RL, L1, L2, and, instead of L3, the weaker $CNpLNLp$, which is equivalent to $CpLMp$ or $CMLpp$ (as L3 itself is equivalent to $CMpLMp$ or $CMLpLp$). Given this result, it is not difficult to show, by induction and the rules for quantifiers, that we have at least the full 'Brouwersche' system for $\Pi n L^n$, i.e. for our defined L. For L2 we have simply

C	(1)	$\Pi n L^n p$	
K	(2)	$L^0 p$	(1, U.I.)
	(3)	p	(2, Df.L^0).

For L1, we clearly have $CL^0 CpqCL^0 pL^0 q$ ($= CCpqCpq$), and since we have $CL^1 CpqCL^1 pL^1 q$, we may prove $CL^{n+1} CpqCL^{n+1} pL^{n+1} q$ from any $CL^n CpqCL^n pL^n q$ as follows:

1.	$CL^n CpqCL^n pL^n q$	(hyp.)
2.	$L^1 CL^n CpqCL^n pL^n q$	(1, RL for L^1)
3.	$CL^1 L^n CpqL^1 CL^n pL^n q$	(2, L1 for L^1)
4.	$CL^1 L^n CpqCL^1 L^n pL^1 L^n q$	(3, L1 for L^1)
5.	$CL^{n+1} CpqCL^{n+1} pL^{n+1} q$	(4, Df.L^{n+1}).

[1] For this abridgement of the definition used in *Past, Present, and Future*, I am indebted to Mr. J. R. Lucas.

Hence we have $\Pi nCL^nCpqCL^npL^nq$, from which quantification theory takes us to

$$C\Pi nL^nCpqC\Pi nL^np\Pi nL^nq,$$

i.e. L1 for our defined L. $CMLpp$ is provable similarly.

But in fact we have not only the 'Brouwersche' system but S5 for this L. For Lemmon has shown that not only in S5 but even in the 'Brouwersche' system, supplemented by the usual rules for quantifiers, we may prove the 'Barcan formula' $C\bar{\Pi}nLf(n)L\Pi nf(n)$. This enables us to prove, for our defined L, the law $CLpLLp$, thus:

1. $C\Pi nL^npL^{m+k}p$	(U.I.)
2. $CL^{m+k}pL^mL^kp$	(provable by induction)
3. $C\Pi nL^npL^mL^kp$	(1, 2)
4. $C\Pi nL^np\Pi k\Pi mL^mL^kp$	(3, $\Pi 2m$, $\Pi 2k$)
5. $C\Pi nL^np\Pi kLL^kp$	(4, Df.L)
6. $C\Pi nL^npL\Pi kL^kp$	(5, Barcan)
7. $CLpLLp$	(6, Df.L).

And it is known that the addition of 7 to the 'Brouwersche' system yields S5. Proofs of L4 and L5 are obvious.

The U-calculus now at last becomes simply a part of tense logic, and the minimal U-calculus is provable from the tense logic K_t, the axioms T8, T9, and T2.2 for the special variables (the M and L which appear in T8 and T2.2 now being seen as tense-logical functors), and quantification theory. From this basis we can in fact prove a little more than the minimal U-calculus; we can also prove a special condition on U, namely

UT5. $\Pi a\Pi b(U \cup \breve{U} \cup I)_* ab$,

i.e. every pair of instants is related by the ancestral of the logical sum of U, its converse and identity. This corresponds to the assumption that the time-series (whatever its shape) is unique, i.e. that all instants may be located from 'now' by some combination of 'will be's and 'has been's. This does not yield any new ordinary tense logical formulae (i.e. without such symbols as our ΠnL^n) preceded by Ta; though it does strengthen the conditions on U that may be proved by adding various tense-logical axioms to K_t. And if we add UT5 to RT, T1–7, and

UT1–4, we may prove not only K_t, T8, and T9 but the equivalence of Lp ($= \Pi a T a p$) to the tense-logical $\Pi n L^n p$, and so all the postulates used in the present section.

At this stage, however, there is no point in having U and T as primitives in addition to G and H. The latter, it should be noted, we *must* have at all stages but the first, for the formulation of UT1 and 2. At the first stage, these last are definitions, and U is our only primitive constant. At the second, we need U, T, G, and H. At the third, we can get rid of U, and at the fourth of T also, leaving only G and H.

7. *Recapitulation.* The stages of our progress from what could be regarded as a pure earlier-and-later logic to what can be regarded as a pure tense-logic may be tabulated as follows (putting in parentheses those postulates which cease to be independent when a later stage is reached):

	T-Postulates	*UT-Postulates*
System I		
	T1. $CTaCpqCTapTaq$	UT1. $ETaGp\Pi bCUabTbp$
	T2.1. $CTaNpNTap$	UT2. $ETaHp\Pi bCUbaTbp$
	T2.2. $CNTapTaNp$.	

System II. Add to System I the following:

	T3. $C\Pi aTapp$	(UT3. $CUabTcUab$)
	(T4. $C\Pi aTapTb\Pi aTap$)	
	T5. $CTapTbTap$.	
	RT: $\vdash\alpha \rightarrow \vdash Ta\alpha$	

Lp definable as $\Pi aTap$.

System III. Add to System II the following:

	T6. Taa	UT4. $CTabCUcaUcb$
	T7. $CTapCap$.	

System equivalent to System IV: Add to System III the following:

UT5. $(U \cup \breve{U} \cup I)_* ab$.

And the stages by which tense-logic so swells as to encompass the earlier-later logic may be tabulated thus (again bracketing those postulates which cease to be independent at a later stage):

System K_t

RG: $\vdash\alpha \rightarrow \vdash G\alpha$ RH: $\vdash\alpha \rightarrow \vdash H\alpha$

A1.1. *CGCpqCGpGq* A1.2. *CHCpqCHpGq*

A2.1. *CPGpp* A2.2. *CFHpp*

P defined as *NHN*, *F* as *NGN*.

System equivalent to System III: Add to K_t the following:

RL: $\vdash\alpha \rightarrow \vdash L\alpha$

L1. *CLCpqCLpLq* L2. *CLpp*

L3. *CNLpLNLp*

L4. *CLpGp* L5. *CLpHp*

T8. *NLNa* T9. *Σaa* T2.2. *ALCapLCaNp*

Tap definable as *LCap*, *Uab* as *LCaFb*.

System IV: Add to the above the following:

Df.L^n: $L^0p = p;\quad L^{n+1}p = KGL^npHL^np$

Df.*L*: $Lp = \Pi nL^np$.

8. *The uniqueness of the time-series.* I want, in conclusion, to look more closely at the beginning and the end of the movement I have sketched.

When the postulates of the tense-logic K_t are no longer regarded as just by-products of definitions in a first-order theory, but substantial assumptions in an independent discipline, it becomes intelligible to question whether they are all true. This possibility I shall not explore here;[1] but there is a converse point, namely that if we do not regard the postulate UT5, in the logic of the earlier-later relation, as a mere by-product of the definition of that relation in tense-logical terms, it becomes intelligible to question *that* postulate, and to wonder whether there are pairs of instants which are *not* connected, even indirectly, by the earlier-later relation. It is certainly intelligible to deny this postulate if we treat instants as genuine objects and the earlier-later relation as undefined; but perhaps we can also do it *without* falling into this 'Platonism' about instants.

Let me lead up to this last possibility by first observing that even if the earlier-later relation is only a logical construction out of tensed facts, we can give a good sense to the assertion that there are an infinity of different *logically possible* time-series. For this could be just a slightly misleading way of saying that

[1] I do explore it in Paper XIII.

not only the futures that have issued from given past states, but the entire course of history, *might* without inconsistency have been different from what it has been and will be, i.e. it *might have been* that we had *d* and then *e* and then *f* instead of *a* and then *b* and then *c*; or *there would be no inconsistency in supposing* that we had *d* and then *e* and then *f*, instead of *a* and then *b* and then *c*. For even if all propositions are tensed, tense logic is not the whole of logic; there is also a logic of such functors as 'There would be no inconsistency in supposing that—'. What we *cannot* say (if 'and then' is tense-logically defined) is that we *do* have a series *a*, *b*, *c*, and also a series *d*, *e*, *f* that is temporally unconnected with it.

Moreover, there is a logic of such functors as 'It appears from a certain point of view that—', and one could therefore give a good sense to talk about an infinity of different 'apparent' time-series. I suspect that the infinity of 'local proper times' which figure in relativistic physics amount simply to what appears from various points of view, or in various 'frames of reference', to be the course of events. And given how the course of events appears from a certain point of view, your relativistic physicist will be able to calculate how it will appear from certain other points of view. He can also indicate what features of the course of events (what temporal orderings of those events) will be common to *all* points of view, and one can work out a 'tense-logic' for that too. (It turns out to be slightly different in the special and the general theories of relativity.) What the relativistic physicists *cannot* calculate from how the course of events appears from certain points of view is how, in all its details, the course of events actually is. It is not clear to me that there is anything surprising or unacceptable in this conclusion, or that we should be driven by it to renounce the use of forms like 'It appears from such-and-such a point of view that *p*', which assume that there is also a plain *p* which is or is not the case. Einstein himself[1] once said to Carnap that

> the problem of the Now worried him seriously. He explained that the experience of the Now means something special for men, something different from the past and the future, but that this important difference does not and cannot occur within physics.

[1] As reported on p. 37 of *The Philosophy of Rudolf Carnap* (ed. Schilpp; The Library of Living Philosophers).

If my interpretation of relativistic physics is correct, he had something there.

We can, all the same, develop the logic of 'points of view' on the basis of a syntax which does *not* thus suggest that there is a 'real' (though only partly knowable) course of events which presents these various systematically related appearances. We might describe this alternative syntax in a very general way as follows: instead of using the plain *p* for a quite impersonal 'It is (really) the case that *p*', we use it for 'It appears (or is the case) from *this* point of view that *p*', or 'It is the case with *this* person or particle that *p*'. That is, the prefix 'It appears from *this* point of view that—' or 'It is the case with *this* person or particle that—' is one which has the same sort of vacuity in this language as 'It is *now* the case that—' has in ordinary tense logic; it does not need to be expressed, but is understood in all that we say. We then describe what appears to be the case from other points of view, or what is the case with other persons or particles, by using quasi-modal operators which take us from 'this' point of view or particle to the other ones, very much as operators like 'It will be the case that—' take us to other 'nows' from 'this' now.[1] They would, I think, be operators corresponding to the 'signal relation' of relativistic physics.[2] Associated with these other points of view or particles are other time-series. These, like 'this' time-series, are just logical constructions out of tensed facts, but they are reached from *our* 'now' not by tenses alone, but by tenses interrupted by the *other* quasi-modal operators that we use for getting from 'this' point of view or particle to the other ones; it is this that would make UT5 unprovable.

A language of this sort would have a solipsist ring to it which I find as hard to take as Platonism about instants, but it would have the advantage of making scientifically unanswerable questions not even askable. This is perhaps one of those many cases where the logician's main philosophical function is to show that there is no escape from one or another of a group of not very palatable alternatives.

[1] A very crude language of this general type is sketched in the next paper.

[2] See R. Carnap, *Introduction to Symbolic Logic and its Applications*, chapter G. More accurately, we would need a conjunction corresponding to Carnap's relation C, in terms of which the signal relation is defined.

XII

QUASI-PROPOSITIONS AND QUASI-INDIVIDUALS

1. *Egocentric logic.* Practitioners of tense logic are often asked the question, 'If you admit as genuine propositions ones whose truth-value depends on *when* they are propounded, why not also admit ones whose truth-value depends on *where* they are propounded, or *by whom*, etc.?'

Why not, indeed? There are many sentences of ordinary speech which have precisely these peculiarities. The truth of 'It is raining here' does depend on where it is uttered, and so does the truth of 'It is raining five miles away' (and more generally of 'It is raining *there*'); and the truth of 'I am sitting down' does depend on who says it, and so perhaps does that of 'Eating bacon and bananas is nice' (= 'I like it') and 'Jones is very tall' (= 'He is much taller than I am'). Nor is it at all difficult to concoct a rigorously formalized language with similar features.

There could, for example, be a language in which there are no proper names or pronouns, but in which the ultimate subject of every sentence is understood to be the speaker. A man might, for example, just say 'Sitting' to indicate that he is himself sitting, i.e. his 'Sitting' would have precisely the force of the English sentence 'I am sitting'. (There could be, perhaps even are, *games* in which ordinary English speakers arranged in a circle report on their activities in this subject-less fashion.) Lacking names and pronouns, the only way that a speaker of this language would have of describing the activities of other people would be by certain 'modalizings' of his sentences which presented the activities of others as being, in a sense, indirect activities of his own. He might, for example, use the form 'All-tall sitting' to mean that everyone taller than himself is sitting, i.e. *their* plain sitting is presented as *his* 'all-tall sitting', i.e. his having-everyone-taller-sitting. This form obviously parallels 'It

will always be that *p*' in tense logic; 'It will always be that Jones is sitting' is equivalent to 'At all instants later than now, Jones is sitting', and his plain sitting at *those* instants is presented as his sitting-for-evermore at *this* instant. 'Some-tall sitting' would be similarly used for '*Someone* taller than me is sitting', and would parallel 'It will be that Jones is sitting' for 'At some instant later than now, Jones is sitting', which presents his sitting at some later instant as his future-sitting at the present instant. 'All-short sitting' and 'Some-short sitting' would analogously express the sitting of everyone and of someone *shorter* than the speaker, just as we use 'It has always been that—' and 'It has been that—' to indicate what goes on at *earlier* instants than the time of speech. Indeed, there is no reason why we should not symbolize 'All-tall *p*', 'Some-tall *p*', 'All-short *p*' and 'Some-short *p*' as *Gp*, *Fp*, *Hp*, and *Pp*, placing these prefixes in the same category as negation, just as we do when they are used for tenses.

Iterated quasi-modalities would of course express oblique obliquities. 'All-tall (some-tall sitting)', *GFp*, would mean 'It is true of everyone taller than me that someone taller than him is sitting', and 'Some-tall (all-tall sitting)', *FGp*, would mean 'It is true of someone taller than me that everone taller than him is sitting'.

Sentences of this 'egocentric' language will not translate as sentences of a normal 'person-neutral' first-order theory, but will be more like the *predicates* of such a theory, the modalities then having the obvious translations (with *Uab* for '*a* is shorter than *b*'):

T1. $(Gp)a = \Pi bCUabpb$
T2. $(Fp)a = \Sigma bKUabpb$
T3. $(Hp)a = \Pi bCUbapb$
T4. $(Pp)a = \Sigma bKUbapb.$

Egocentric truth-functions (asserted by *a*) analogously translate by

T5. $(\mathcal{N}p)a = \mathcal{N}(pa)$
T6. $(Cpq)a = C(pa)(qa).$

These 'translations', with ordinary quantification theory, will suffice to guarantee that 'egocentric logic' includes the following

laws (i.e. that all substitution-instances in the following will be true whoever says them):

T7. *CGCpqCGpGq*,

e.g. if all-tall if-standing-then-uncomfortable, then if all-tall standing, all-tall uncomfortable;

T8. *CpGPp*,

e.g. if sitting then all-tall some-short sitting, or in English, 'If I am sitting, then it is true of anyone taller than me that someone shorter than him is sitting'; and of course their images

T9. *CHCpqCHpHq*
T10. *CpHFp*.

We will also have the rules

RG, RH: $\vdash\alpha \rightarrow \vdash G\alpha$; $\vdash\alpha \rightarrow \vdash H\alpha$,

i.e. what is logically true of me is logically true of everyone taller and of everyone shorter than me (e.g. not only I, but everyone taller than me, and everyone shorter than me, sits if he sits and stands if he stands). Further, because the relation of being shorter than is transitive (*CUabCUbcUac*) we have the law

T11. *CFFpFp*,

e.g. if someone taller than me has someone taller than him sitting, then someone taller than me is sitting.

There is, in short, a pretty detailed formal parallel between tense logic and this rather simple type of 'egocentric logic'. In fact, we can obtain something *more or less* like tense logic if we take *any* first-order theory whatsoever, treat its one-place predicates as if they were propositions, and treat an $(n+1)$-place function with n predicate-arguments and one individual argument as an n-place function of propositions. That is, we equate something of the form

$$f(p_1, p_2, \ldots, p_n, a) \tag{1}$$

with something of the form

$$(f'(p_1, p_2, \ldots, p_n))a \tag{2}$$

and drop the *a*. Our (1) could be, for example, *ΠbCUabpb*, which is a function of the one predicate *p* and the one name *a*

(the other variable, *b*, is bound, and the remaining symbols are constants), and then we use *G* for the resulting *f'* in (2). The formal manipulation is the same whatever the variables stand for, and whatever predicate-constants we employ in constructing the function *f*.

2. *Person-propositions.* An egocentric propositional logic with quasi-modal operators is a strange form for the theory of comparative height to take. Still, as we have just seen, it *can* be done that way. Does not this render completely trivial the fact that the theory of the earlier-later relation can be set out in this way too? We may well be inclined to say: *of course* the theory of the earlier-later relation can be developed by using quasi-modalities (tenses), for the theory of *any* relation can be developed that way; but why do it, either in this case or in any other?

One answer to this which we might think of making is that we can not only develop tense logic within the first-order theory of the earlier-later relation, but can equally develop this first-order theory within tense logic (enriched by a little higher-order quantification). For we can identify an instant with a tensed proposition, namely with the conjunction of everything that would ordinarily be said to be true *at* that instant; or alternatively, with something that would ordinarily be said to be true at that instant only. We can then interpret being true at an instant as being necessarily or omnitemporally implied by that instant (considered as a proposition), and one instant's being earlier than another as the futurity of the latter being true 'at' the former, i.e. with the former's necessarily or omnitemporally implying that the latter will be the case. The theory of the earlier-later quasi-relation will then become a part of tense logic rather than vice versa. (I say quasi-relation because if *a* and *b* are sentential rather than individual variables, the link in the form '*a* is earlier than *b*' is a two-place *sentential connective* rather than a two-place *predicate* expressing a relation between objects.)[1]

Does not the possibility of this reversal mark off tense logic from other calculi that may be presented as by-products of first-order theories? Or can this reversal be equally performed in these other cases too? I am afraid the answer is that it can. Consider again our 'egocentric' version of the theory of the Tall

[1] Cf. Paper XI.

and the Short. Not only can this be 'explicated' as a by-product of the normal version of this theory, but the latter can also be 'explicated' as a by-product of the former. We can, to start with, identify a person with, or replace him in our system by, the conjunction of all the propositions that would ordinarily be said to be true *of* this person, or by some proposition that would ordinarily be said to be true of him alone. We can then interpret being true 'of' a person as being necessarily or 'omnipersonally' implied by him. And we can say that one person is shorter than another (i.e. we can define the relation U of the associated first-order theory) by saying that the former necessarily or omnipersonally implies some-tall the latter, i.e. that someone taller than him is the latter. These last definitions no doubt require a little explanation, but it is easy enough to give it.

The form '*a* omnipersonally implies that *p*' is true if the egocentric statement 'If *a* then *p*' is true whoever says it. Suppose, for instance, that *a* is the only person sitting, and that he is also drinking (others may be drinking too). We can in these circumstances identify the person *a* with the proposition 'Sitting' (i.e. 'I am sitting'); and 'If sitting then drinking', i.e. 'If I am sitting then I am drinking', will be true whoever says it (true if *a* says it because he is drinking, and true if anyone else says it because he is not sitting—*ex hypothesi*, *a* is the only person doing that). So in these circumstances sitting omnipersonally implies drinking; and this ('Omnipersonally if sitting then drinking') is the proposition of our noun-and-pronounless language by which we may translate the ordinary '*a* is drinking'. Again, if *b* is the only person smoking, and *a* is still the only person sitting, and *b* is shorter than *a*, then 'If I am smoking then someone taller than me is sitting' will be true whoever says it (true if *b* says it because *a* is sitting, and true if anyone else says it because anyone else is not smoking); i.e. smoking will omnipersonally imply some-tall sitting. And this ('Omnipersonally if smoking then some-tall sitting') is how we express '*b* is shorter than *a*' within this language. (With L for 'omnipersonally', $Uba = LCbFa$.)

Finally, the mode 'omnipersonally' itself, i.e. 'It is true of all persons that—', is definable in terms of the modalities or quasi-tenses 'All-tall' and 'All-short', though we must construct this definition carefully. It will not do to define 'It is true of

everyone that *p*' simply as '*p* and all-tall *p* and all-short *p*', e.g. 'All-persons sitting' as 'I am sitting and so is everyone taller than me and so is everyone shorter'. For this does not cover persons of the same height as the speaker who are not identical with him. These can be brought in, however, if the defining formula is revised by the addition of 'All-tall all-short *p* and all-short all-tall *p*', i.e. 'Everyone taller than me has *p* being true of everyone shorter than him, and everyone shorter than me has *p* being true of everyone taller than him'. (This works, at least, so long as people are of at least two different heights.) At this point 'egocentric logic' is rather like the tense logic that might be extracted from the earlier-later relation of the special theory of relativity. (In that theory, 'now' is always 'here-now', and what 'is the case here-now and throughout the absolute past and throughout the absolute future' will not necessarily be the case throughout the whole of space-time; but what also 'is the case throughout the whole absolute past of every bit of the absolute future and throughout the whole absolute future of every bit of the absolute past', *will* be true throughout the whole of space-time. The point-instants that are not here-now but are not absolutely past or future either, are like the persons who are neither taller nor shorter than nor yet identical with the speaker.)[1]

Given this definition, in terms of our basic modalities, of 'omnipersonally', it is possible to state within the system what it is for a given proposition to be a person, or at all events what it is for a given proposition to be an individual (i.e. an object of which it makes sense to say that it is taller or shorter than another object). The first requirement is that for every egocentric proposition *q*, either the given proposition *p* omnipersonally implies that *q*, or it omnipersonally implies that not *q*. (This is how we state in Egocentric that every individual is either sitting or not sitting, either drinking or not drinking, either smoking or not smoking, etc.) But this criterion alone would not exclude propositions which apply to *no* individual, since these would omnipersonally imply *every* proposition. (The proposition 'Smoking and not smoking', for example, omnipersonally implies every proposition; i.e. 'If I am both smoking and not smoking then *p*' is true whoever says it, and whatever *p* may

[1] Cf. *Past, Present, and Future,* Appendix B, Section 5.

be.) So our second condition—for a proposition to be an individual—must be that it must *not* omnipersonally imply every proposition. Symbolically, if we write Qp for 'p is an individual', we define Q thus:

$$Lp = KKKKpGpHpGHpHGp$$
$$Qp = K\Pi qALCpqLCpNqN\Pi qLCpq.$$

If we introduce special propositional variables a, b, c, etc. for p's such that Qp, and write $LCap$ as pa and $LCaFb$ as Uab, we can reconstruct the ordinary first-order theory of the Tall and the Short within the quasi-modal egocentric system.

3. *Formalism and ontology.* Philosophically, where do we go from here? We *could* turn the tables on the objectors to tense logic by saying that not only are 'instants' not genuine individuals but there are *no* genuine individuals, only certain propositions that can be formally treated *as if* they were individuals. I suspect that there would be fewer takers for this theory than for the theory that instants, or point-instants, *are* genuine individuals; though some of the things that Leibniz said suggest that he did think of a 'monad' as the conjunction of all the propositions that would ordinarily be said to be true of it. I remember, too, C. A. Meredith remarking in 1956 that he thought the only genuine individuals were 'worlds', i.e. propositions expressing total world-states, as in the opening of Wittgenstein's *Tractatus* ('The world is everything that is the case'). Or 'egocentric logic' might be thought of as formalizing the strange remarks that are made about solipsism in the *Tractatus* 5. 62–5. 6331 ('What the solipsist *means* is quite correct; only it cannot be *said*'. 'I am my world'. 'The subject does not belong to the world: rather, it is a limit of the world').

But why go to extremes, in either direction? The physicists' equation

$$PV = R\theta,$$

relating the pressure, volume, and temperature of a gas, can be used either to raise the temperature of a given volume of gas by increasing the pressure on it, or to increase the pressure it exerts by heating it; there is nothing in the equation that compels us to do either of these rather than the other, or prevents

us from sometimes doing one and sometimes the other. Similarly with the philosophical use of the logical equation-form of Section 1,

$$f(p_1, p_2, \ldots, p_n)a = (f'(p_1, p_2, \ldots, p_n))a.$$

So far as I can see, there is nothing philosophically disreputable in saying that (i) persons just *are* genuine individuals, so that their figuring as individual variables in a first-order theory needs no explaining (*this* first-order theory being, on the contrary, the only way of giving sense to its 'modal' counterpart), whereas (ii) instants are *not* genuine individuals, so that *their* figuring as values of individual variables *does* need explaining, and it is the related 'modal' logic (tense logic) which gives to the first-order theory what sense it has.

What I am propounding here is an alternative to Quine's account of what he calls 'ontological commitment'. The 'entities' which we 'countenance' in our 'ontology' do *not* depend, as Quine says they do, on what kinds of variables we are prepared to bind by quantifiers. They depend on what variables we take seriously as individual variables in a first-order theory, i.e. as subjects of predicates rather than as *assertibilia* which may be qualified by modalities. If we prefer to handle instant-variables, for example, or person-variables, as subjects of predicates, then we may be taken to believe in the existence of instants, or of persons. If, on the other hand, we prefer to treat either of these as *propositional* variables, i.e. as arguments of truth-functions and of modal functions, then we may be taken as *not* believing in the existence of instants, etc. (they don't exist; rather, they are or are not the case). To use another of Quine's phrases, ontological commitment varies inversely with modal involvement.

4. *The two problems of ontology.* This is not the whole story, however. Philosophers worry about 'ontology' for two main reasons. In the first place, they may be worried about the *abstractness* of certain alleged individuals or objects, e.g. instants or events. They are compelled to admit that there certainly are truths which appear to be about such objects, and yet hesitate to say that they really *are* objects in the sense in which things and persons are, or that they 'exist' in the sense in which things and persons do. It is in this context that 'logical grammar', of the

sort that we have just been doing, can be helpful; we can show that certain apparent names need not be seriously regarded as names of objects, by producing paraphrases in which such names are dispensed with in favour of other parts of speech, or it may be of whole sentences.

But in the second place, philosophers may be worried about the fact that certain undoubted truths appear to be about objects which, though not in the least abstract, are merely fictitious, or are mere has-beens or will-be's (i.e. they have ceased to exist, or have not yet begun to). Quantification, it seems to me, *is* relevant to *this* worry, but not quite in the way Quine says it is; indeed in almost exactly the opposite way. Quine says in effect that non-existents cannot figure as the values of bound variables; I would suggest that, on the contrary, this is the only way in which non-existents of this sort *can* figure. I cannot directly refer to what does not exist but is merely imagined to exist, or is merely going to exist; but I *can* make purely *general* (i.e. quantified) statements about the imaginary or future denizens of the world. The quantification, however, must occur within a 'modality'. I may, indeed, imagine some real object to be a mermaid; we can then say that there is an x such that x is imagined by me to be a mermaid; but if what is involved is, as we say, a 'merely imaginary' mermaid, then we cannot say that *there is an x such that I imagine that x* is a mermaid, but only that *I imagine that there is an x such that x* is a mermaid.[1] Analogously, it may be that some existing person is going to live so long as to rule England in A.D. 3000; more likely, however, the ruler of England at that date does not yet exist, in which case we cannot say that *for some x, it will be the case then that x* rules England, but only that *it will be the case then that for some x, x* rules England.

I want to say, in short, that 'It will be the case that something ϕs' does not entail 'There is something of which it will be the case that it ϕs', i.e. that $CF\Sigma x\phi x\Sigma xF\phi x$ is not a law. But if we subjoin ordinary quantification theory to the tense logic defined by the theses T7-10 and the rules RG, RH of our first section, we obtain $CF\Sigma x\phi x\Sigma xF\phi x$ as a theorem. The tense logic thus defined—the system called by E. J. Lemmon K_t—is, however, the weakest that we can obtain if we treat this logic

[1] Cf. G. E. Moore, *Commonplace Book*, pp. 243–5.

as a by-product of a first-order theory of the earlier-later relation (it is obtainable without assuming *anything* about the character of that relation). Isn't *this*, at last, a sure-fire formal proof that tense logic is *not* a by-product of a first-order theory of the earlier-later relation? It approaches that, I think; but of course its underlying account of the relations between quantification and ontology won't convince everyone. There are always answers.

XIII

TENSE LOGIC FOR NON-PERMANENT EXISTENTS

1. *Tense logic, earlier-later logic, and modal logic.* By a 'tense logic' I mean a calculus in which the variables *p*, *q*, *r*, etc. stand for 'propositions' which may be true or false at different times, and in which the usual two-valued truth-functions *Cpq*, *Apq*, *Kpq*, *Epq*, *Np*, etc. are supplemented by the two forms *Fp* for 'It will be the case that *p*' and *Pp* for 'It has been the case that *p*'. The forms *Gp*, for 'It will always be the case that *p*', and *Hp*, for 'It has always been the case that *p*', are generally taken to be definable in terms of the others as *NFNp* and *NPNp* respectively.

It is sometimes argued that the 'propositions' of such a system are not properly so described, being in fact *predicates* of instants, its characteristic functors being used not to form propositions from propositions but to form predicates from predicates. We might use the variables *a*, *b*, *c*, etc. to stand for instants, and say that the theses of tense logic are all to be understood as predicable of any arbitrary instant *a*. We then write *pa* for '*a* is a *p*-ish instant' or 'The pseudo-proposition *p* is true at the instant *a*', and read $(Fp)a$, $(Pp)a$, $(Cpq)a$, and $(Np)a$ similarly. If we use script letters for truth-functors of which the arguments are genuine propositions, and Σa for an existential quantifier binding instant-variables, we might define our principal predicate-formers as follows:

$$\begin{aligned}(Cpq)a &= \mathscr{C}paqa\\ (Np)a &= \mathscr{N}pa\\ (Fp)a &= \Sigma b\mathscr{K}Uabpa\\ (Pp)a &= \Sigma b\mathscr{K}Ubapa,\end{aligned}$$

where *U* is a two-place predicate which may be read informally as '—is earlier than—'. The last two definitions assert that 'It is true at *a* that it will be the case that *p*' and 'It is true at *a* that it has been the case that *p*' respectively mean

that p is true at some instant later, and that it is true at some instant earlier, than a.

If we incorporate these definitions in an appropriate first-order theory, we may prove that certain tense-logical theses are predicable of ('true at') any arbitrary instant a. E. J. Lemmon has found that the tense-logical theses which have this property are precisely those which are derivable by substitution, detachment, and the rules

$$\vdash \alpha \rightarrow \vdash \mathcal{N}F\mathcal{N}\alpha, \qquad \vdash \alpha \rightarrow \vdash \mathcal{N}P\mathcal{N}\alpha,$$

from some basis for propositional calculus and the four further axioms

1.1. *CNFNCpqCFpFq* 2.1. *CFNPNpp*
1.2. *CNPNCpqCPpPq* 2.2. *CPNFNpp.*

The tense logic defined by these postulates he called K_t. If certain conditions are imposed on the relation U, richer tense logics are obtainable (as predicable of an arbitrary instant a). For example, if we impose on U the following three conditions (where I = identity):

CUabCUbcUac (transitivity)
CUabCUacAAUbcUcbIbc (non-branching in the future)
CUbaCUCaAAUbcUcbIbc (non-branching in the past),

we may prove (as predicable of any arbitrary a) all the theses (and only the theses) of the tense logic obtained by adding to K_t the three further axioms

3. *CFFpFp*
4. *CPFpAApFpPp*
5. *CFPpAApFpPp.*

In this strengthened system, if we define *Mp* as *AApFpPp*, we obtain for this M the modal logic S5, which Lemmon[1] has shown to be the system obtainable by subjoining to propositional calculus, with substitution and detachment, the axiom

CpMp

[1] E. J. Lemmon, 'Alternative Postulate-sets for Lewis's S5', *Journal of Symbolic Logic*, vol. 21, no. 4 (December 1956), pp. 347–9.

and the rule (call it RCM)

$\vdash C\alpha\beta \rightarrow \vdash CM\alpha\beta$, if every variable in α falls within the scope of an M.

(Lemmon proves this for slightly different postulates, the equivalence of which to these ones is obvious.)

2. *Modification of modal logic for contingent existents.* If we adopt a rather different view of tense logic from that sketched in the previous section, and treat its 'propositions' as being just as properly so-called as those of the earlier-later calculus, it is possible to derive the latter from tense logic rather than vice versa. Moreover, if the postulates of the 'minimal' tense logic K_t are treated not as by-products of definitions in a first-order theory but as substantial assumptions, it is intelligible to question their truth. This is done, for example, in chapter VIII of *Past, Present, and Future.* The detailed arguments need not be reproduced here, but their upshot is as follows: when we introduce individual name-variables, and predicates attaching to them, it is arguable that before and after an individual x exists there are no such propositions as ϕx (though there may be propositions to the effect that there has been or will be *some* individual with such-and-such properties, viz. those of x). That is the way the matter is put by G. E. Moore;[1] I have myself sometimes put it by saying that at the times in question no such proposition is 'statable'. This latter locution has the disadvantage of suggesting that the difficulty here is simply with our mechanisms of reference; I want to say rather that there are no *facts* about x to be stated except when x exists. Moore's terminology, and my own alternative one about 'facts', has the disadvantage of suggesting that there are abstract entities called 'facts' and/or 'propositions' which exist as individuals do. The terminology does not matter, so long as the misleading suggestions are removed.

Formally, this line of argument makes it no longer plausible to identify Gp, 'It will always be the case that p', with the mere $NFNp$, 'It will never be the case that not p', or H with NPN; and it would seem that although we have the rules

$$\vdash \alpha \rightarrow \vdash NFN\alpha, \qquad \vdash \alpha \rightarrow \vdash NPN\alpha,$$

[1] G. E. Moore, 'Necessity', in *Lectures on Philosophy* (1966), pp. 129–31.

we do not have

$$\vdash \alpha \rightarrow \vdash G\alpha, \qquad \vdash \alpha \rightarrow \vdash H\alpha;$$

and conversely, although we have *CGCpqCFpFq* and its image, we do not have *CNFNCpqCFpFq* or its image. And although we have *CFNPNpp*, *CFHpp*, and their images, we do not have *CNGNHpp*, *CNGPNpp*, or their images.

The first problem I wish to pose is simply that of giving postulates for a minimal tense logic which will allow for this complicating factor. But before making a proposal here, I would observe that a partly similar problem arising in modal logic has already been solved. If we have individual name-variables in modal logic, it is arguable that in possible worlds in which the individual x does not figure, no proposition of the form ϕx can figure either, and this makes a difference between *Lp*, 'Necessarily *p*', i.e. 'It is the case in all possible worlds that *p*', and *NMNp*, 'Not possibly not-*p*', i.e. 'It is not the case in any possible world that not-*p*'. In 1959[1] I gave some postulates which I conjectured would suffice for a modal logic which takes account of this possibility, and in 1964,[2] by drawing upon a result of R. A. Bull,[3] I proved this conjecture correct. The modal logic in question is called Q, and its axiomatization is as follows: beside the ordinary modal form *Mp*, we introduce a form *Sp*, which may be taken as meaning 'In all possible worlds there is such a proposition as *p*', and we define *Lp* as *KSpNMNp* (*Sp* then becomes provably equivalent to *LCpp*). For *S* we have the rules

RS1. $\vdash CS\alpha Sp$, for any p occurring in α
RS2. $\vdash CSpCSq\ldots S\alpha$, where p, q,...are all the variables in α.

For *M* we have the axiom *CpMp* and the following modification of the rule given in the last section for S5:

RSM: $\vdash C\alpha\beta \rightarrow \vdash CSpCSq\ldots CM\alpha\beta$, where all the variables in β fall within the scope of an *M* or an *S*, and p, q,... are all the variables in β that are not in α.

[1] A. N. Prior, 'Notes on a Group of New Modal Systems', *Logique et Analyse*, no. 6–7 (Apr. 1959), pp. 122–7.

[2] A. N. Prior, 'Axiomatisations of the Modal Calculus Q', *Notre Dame Journal of Formal Logic*, vol. 5, no. 3 (July 1964), pp. 215–17.

[3] R. A. Bull, 'An Axiomatisation of Prior's Modal Calculus Q', ibid., pp. 212–14.

In the following sections, by introducing functors analogous to *S*, I shall propose modified postulates for the minimal tense logic K_t, and prove that if we strengthen this by adding suitable postulates for time's linearity and the transitiveness of the earlier-later relation, and again define *Mp* as *AApFpPp*, and *Sp* in terms of the analogous functors, we obtain for this *M* the modal system Q.

3. *Revised minimal tense logic.* We introduce, beside *F* and *P*, the functors *T* and *Y* (from 'tomorrow' and 'yesterday') such that *Tp* is true if and only if there will always be, and *Yp* if and only if there always has been, such a proposition as *p*. For these we need, to begin with, postulates analogous to RS1 and 2, expressing the notion that a complex proposition is 'statable' or 'there' if and only if all of its components are. That is, we have these schemata:

T1: $\vdash CT\alpha Tp$, where p is any variable in α
T2: $\vdash CTpCTq\ldots T\alpha$, where $p, q, \ldots$ are all the variables in α;
Y1: $\vdash CY\alpha Yp$, where p is any variable in α
Y2: $\vdash CYpCYq\ldots Y\alpha$, where $p, q, \ldots$ are all the variables in α.

We do not need any special form for 'It is *now* statable that *p*' or 'There is *now* such a proposition as the proposition that *p*', since this is tautologous, being itself only statable because *p* is. (Our whole logic, for this reason, must be developed in terms of what is now statable. To quote from an earlier article,[1] 'Nothing can be surer than that whereof we cannot speak, thereof we must be silent, though it does not follow from this that whereof we could not speak yesterday, thereof we must be silent today'.)

For *F* and *P* alone we have

PF: *CPNFNpp* RF: $\vdash\alpha \rightarrow \vdash NFN\alpha$
FP: *CFNPNpp* RP: $\vdash\alpha \rightarrow \vdash NPN\alpha$.

For the combination of the two types of operators we need at least the following:

Df. *G*: $Gp = KTpNFNp$ Df. *H*: $Hp = KYpNPNp$
FT1: $\vdash CTpCTq\ldots CNFNC\alpha\beta CF\alpha F\beta$
PY1: $\vdash CYpCYq\ldots CNPNC\alpha\beta CP\alpha P\beta$,
where $p, q\ldots$ are all the variables in β that are not in α.

[1] A. N. Prior, 'Thank Goodness that's Over', *Philosophy*, vol. 34, no. 128 (Jan. 1959), pp. 11–17.

We need more than this, but before laying down further postulates we had better clarify some ambiguities in the forms Tp and Υp. Remember that we are at this stage trying to set up a *minimal* system, in which it is not assumed, for example, that time is infinite both ways, or that it is strictly linear. Consequently we must give a sense to Tp in the case in which we are at the end of time and there is *no* future, and also in the case in which we stand at a cross-roads and there is no such thing as *the* future but only a number of *alternative* futures, and similarly for Υp and the past. It is most convenient to use the Boolean 'always', making Tp vacuously true if there is no future; and in the matter of branching, it is convenient to take Tp as meaning that p will be statable throughout *all possible* futures; and analogously with Υp. Fp is simply false if there is no future, and true if it will be the case in *some* possible future that p; and Pp analogously.

Given this interpretation of our operators, we may consider various candidates for theses. $CTpFTp$ is obviously excluded, since at the last moment of time, if there were one, Tp would be vacuously true and FTp false. The converse $CFTpTp$ would be vacuously *true* (because of the falsehood of its antecedent and the truth of its consequent) at the end of time; but with a branching future it could be false if p were of the form ϕx and x might or might not be going to exist for ever, for this would mean that Tp is true in some possible futures (giving us FTp) but not in all (depriving us of the consequent Tp). If, on the other hand, there is only one future, $CFTpTp$ is a reasonable thesis if we exclude the possibility that x, which must exist now for p ($= \phi x$) to be now statable, may go out of existence and then start to exist again, this time for ever. (The realization of this possibility *would* give us FTp without Tp, even in linear time.) So we might wish to have $CFTpTp$ in a calculus for linear time, though not in a minimal one. And even *without* excluding the possibility of intermittent existence,[1] we would have in linear time the weaker thesis $CFKTp\Upsilon pTp$.

Consider now the pair $CTpPTp$, $CPTpTp$. There seems nothing to be said for the former, on any hypothesis; if x now exists for the first time, but will exist from now on, we have

[1] For a discussion of this possibility, see *Analysis* Problem no. 11, in *Analysis*, vol. 17, no. 6 (June 1957).

$T\phi x$ but not $PT\phi x$. On the other hand, $CPTpTp$ seems certainly valid. Even with branching time, if it was true in some possible past that p would be statable throughout all futures then possible, it must be statable throughout all futures now possible.

Similar considerations apply to the group $CYpPYp$, $CPYpYp$, $CYpFYp$, $CFYpYp$.

In sum, we have the following two postulates to add to the basis set out above:

FY: $CFYpYp$
PT: $CPTpTp$.

This whole system, subjoined to propositional calculus with substitution and detachment, I shall call QK_t. It clearly collapses to K_t if we add the postulates $\vdash Tp$, $\vdash Yp$, or define both forms as Cpp.

4. *Deductions in QK_t.* Examples of the schemata FT1 and PY1 include the following pair:

1. $CNFNCKpqpCFKpqFp$
2. $CNPNCKpqpCPKpqPp$.

(Here, there are *no* variables in β that are not in α.) From $CKpqp$, RF and 1, and from $CKpqp$, RP and 2, we obtain

6. $CFKpqFp$
7. $CPKpqPp$.

(Numbers 3, 4, and 5 are already attached to some theses in Section 1 to which we shall later be returning.) For a more complicated proof, we have the following:

8.	$NFNCNCCpqqNp$	(RF applied to $CNCCpqqNp$, a transposition of $CpCCpqq$)
9.	$CFNCCpqqFNp$	(8, FT1)
10.	$CNFNpNFNCCpqq$	(9, $CCpqCNqNp$)
11.	$CNFNpCFCpqFq$	(10, FT1)
12.	$CFNNpFp$	($CNNpp$, RF, FT1)
13.	$CNFpNFNNp$	(12, $CCpqCNqNp$)
14.	$CNFpCFCNpqFq$	(13; 11 p/Np)
15.	$CFCNpqCNFpFq$	(14, $CCpCqrCqCpr$)
16.	$CFApqAFpFq$	(15, $A = CN$)
17.	$CPApqAPpPq$	(analogously).

The converses of 16 and 17, however, are not theorems. The basic reason is that the formulae

$$CNFNCpApqCFpFApq, \qquad CNPNCpApqCPpPApq$$

are not instances of FT1 and PY1 unless they are respectively preceded by CTq and CYq; hence we cannot use them (with RF, RP, and $CpApq$) to prove $CFpFApq$, $CPpPApq$; we can only obtain $CTqCFpFApq$ and $CYqCPqPApq$. (A counter-example to the simple $CPpPApq$ would be 'If it has been that God alone exists, then it has been that either God alone exists or I don't exist'; here the antecedent, on the Christian hypothesis is true; but there is only such a proposition as 'Either God alone exists or I don't exist' when I do exist, both its disjuncts being then false, so this disjunction has never been true.)

However, FT1 and PY1 do enable us to lay down the simple

18. $CNFNC\alpha\beta CF\alpha F\beta$
19. $CNPNC\alpha\beta CP\alpha P\beta$

for all those α's and β's which contain the same variables. For such cases, therefore, we can deduce

20. $CF\alpha FA\alpha\beta$ ($CpApq$, RF, 18)
21. $CF\beta FA\alpha\beta$ ($CqApq$, RF, 18)
22. $CAF\alpha F\beta FA\alpha\beta$ (20, 21, $CCprCCqrCApqr$).

(We also have $CAP\alpha P\beta PA\alpha\beta$ analogously from 19.)

With this restriction, in fact, 18 and 19, with RF and RP, yield the rules

RFC: $\vdash C\alpha\beta \rightarrow \vdash CF\alpha F\beta$
RPC: $\vdash C\alpha\beta \rightarrow \vdash CP\alpha P\beta$.

If we now introduce the form Mp as an abbreviation for $AApFpPp$, we may immediately prove the rule

RM: $\vdash \alpha \rightarrow \vdash NMN\alpha$.

For the consequent of this amounts to $\vdash NAAN\alpha FN\alpha PN\alpha$, which by de Morgan equates to $\vdash KKNN\alpha NFN\alpha NPN\alpha$, and we may prove each conjunct of this by $CpNNp$, RF, and RP. Again, for α's and β's which contain the same variables we have

23. $CNMNC\alpha\beta CM\alpha M\beta$,

that is

$$CNAANC\alpha\beta FNC\alpha\beta PNC\alpha\beta CAA\alpha F\alpha P\alpha AA\beta F\beta P\beta,$$

i.e. by de Morgan

$$CKKC\alpha\beta NFNC\alpha\beta NPNC\alpha\beta CAA\alpha F\alpha P\alpha AA\beta F\beta P\beta.$$

This is of the form $CKK\alpha\beta\gamma CAA\delta\epsilon\xi AA\phi\psi\chi$, where we have $C\alpha C\delta\phi$ (by $CCpqCpq$), $C\beta C\epsilon\psi$ (by 18) and $C\gamma C\xi\chi$ (by 19), and we have from p.c.

$$CCpCqrCCsCtuCCvCwxCKKpsvCAAqtwAArux.$$

From RM and 23 (where α and β have the same variables) we obtain the rule (with the same restriction)

RMC: $\vdash C\alpha\beta \rightarrow \vdash CM\alpha M\beta$.

This rule is freely used throughout the following series of proofs, where none of the theses contains more than one distinct variable:

24. $CpMp$	(= $CpAApFpPp$, from p.c.)
25. $CFNAApFpPpFKKNpNFpNPp$	(de Morgan, RFC)
26. $CFNAApFpPpFNPp$	(25, 6)
27. $CFNMpFNPp$	(26, Df.M)
28. $CPNNpPp$	($CNNpp$, RPC)
29. $CNPpNPNNp$	(28, $CCpqCNqNp$)
30. $CFNPpFNPNNp$	(29, RFC)
31. $CFNMpNp$	(27, 30, FP)
32. $CPNMpNp$	(analogously)
33. $CNMpNp$	(24, $CCpqCNqNp$)
34. $CAANMpFNMpPNMpNp$	(33, 31, 32, p.c.)
35. $CMNMpNp$	(34, Df.M).

We also have, without any restrictions on α and β, the following schema:

SM: $CSpCSq...CNMNC\alpha\beta CM\alpha M\beta$, where p, q...are all the variables in β that are not in α.

For given that $Sp = KTpYp$, we may prove this from FT1 and

PY1 by the same steps that were used in proving 23 from 18 and 19, but with added antecedents. And from this we may prove

36. *CMApqAMpMq*

by steps analogous to those used in proving 16 and 17 from FT1 and PT1.

Further, the definition of *S*, and the p.c. laws of *C* and *K*, obviously take us from T1 and Y1 to RS1, and from T2 and Y2 to RS2.

5. *Deductions in QK_t with added assumptions.* I now want to consider what further theses, particularly of a 'modal' sort, are obtainable when we add to K_t the postulates 3, 4, and 5 of Section 1, for the transitivity of the earlier-later relation and the linearity of time. We may note first that from 3 (*CFFpFp*) and QK_t we may deduce the mirror-image of 3, i.e.

37. *CPPpPp.*

Lemmon noticed that this was deducible from 3 given K_t, and the proofs go through in QK_t also (basically because *PPp* and *Pp* contain the same variables, namely the one variable *p*). Given this, we have the following:

38. *CFpAApFpPp*	(p.c.)
39. *CFFpAApFpPp*	(3, 38)
40. *CPFpAApFpPp*	(4)
41. *CAAFpFFpPFpAApFpPp*	(38, 39, 40, p.c.)
42. *CFAApFpPpAApFpPp*	(16, 41)
43. *CFMpMp*	(42, Df.*M*)
44. *CPMpMp*	(analogously, using 37 and 5)
45. *CAAMpFMpPMpMp*	(*Cpp*, 43, 44)
46. *CMMpMp*	(45, Df.*M*)
47. *CNMpNMMp*	(46, *CCpqCNqNp*)
48. *CMNMpMNMMp*	(47, RMC)
49. *CMNMpNMp*	(48, 35 *p*/*Mp*)
50. *CAMNMpMMqANMpMq*	(49, 46, p.c.)
51. *CMANMpMqANMpMq*	(36, 50)
52. *CMCMpMqCMpMq*	(51, $C = AN$).

We may now consider new theses in T and Y and also in S (where $Sp = KTpYp$). But at this point it seems necessary to add new *postulates* involving these functions which become plausible when time is taken to be linear, namely the pair

FT2. $CFKTpYpTp$
PY2. $CPKTpYpYp$.

It is possible that these are provable when 3, 4, and 5 alone are added to QK_t, but I have not been able to prove them. It is also possible that QK_t is incomplete for its purpose, and that there ought to be further 'non-committal' axioms added to it which *would* yield FT2 and PY2 when 3, 4, and 5 are added. It would certainly be preferable to have FT2 and PY2 as a simple by-product of the addition of 3, 4, and 5. There is, however, no guarantee that they *can* be so derived, and there is independent evidence that making further assumptions about time may require the addition of special postulates involving T and Y. For example, one normal way of expressing the assumption that time will have no end is to lay down $FCpp$ as an axiom. From our present point of view, however, this will not do; for even if time *is* infinite in the future direction, i.e. even if there *is* going to be a future, $FCpp$ could be false because p has just reached the end of its statability. Something like $CTpFCpp$ seems to be required here.

Given FT2 and PY2, the following proof becomes possible:

53.	$CFKTpYpTp$	(FT2)
54.	$CFKTpYpFYp$	($CKpqq$, RF, FT1)
55.	$CFKTpYpYp$,	(54, FY)
56.	$CFKTpYpKTpYp$	(53, 55, p.c.)
57.	$CFSpSp$	(56, Df. S)
58.	$CPSpSp$	(analogously)
59.	$CMSpSp$	(Cpp, 57, 58, Df.M).

We are now in a position to prove the rule RSM of the system Q. (We have already obtained RS1, RS2, and the axiom $CpMp$ in the last section, so this will complete the proof of Q in our enriched QK_t.) We shall adapt to this purpose the method used by Lemmon to prove the postulates given in Section 1 for S5, from a more usual basis.

Firstly, we prove the lemma that where every variable in α falls within the scope of an M or an S, we have $\vdash C M \alpha \alpha$. The simplest way in which the condition that every variable falls within the scope of an M or an S may be met, is by α to be itself of the form $M\beta$ or $S\beta$. In the first case $CM\alpha\alpha$ will be of the form $CMM\beta M\beta$, which is a theorem by 46, and in the second case it will be of the form $CMS\beta S\beta$, which we have by 59. More complicated cases arise either by α being of the form $N\beta$, where β meets the condition, or by its being of the form $C\beta\gamma$, where β and γ meet the condition. So we need to prove (i) that if we have $\vdash CM\beta\beta$ we also have $\vdash CMN\beta N\beta$, and (ii) that if we have $\vdash CM\beta\beta$ and $\vdash CM\gamma\gamma$ we also have $\vdash CMC\beta\gamma C\beta\gamma$. For the first, we proceed thus:

60. $CM\beta\beta$	(hypothesis)
61. $CN\beta NM\beta$	(60, $CCpqCNqNp$)
62. $CMN\beta MNM\beta$	(61, RMC)
63. $CMN\beta NM\beta$	(62, 49)
64. $CMN\beta N\beta$	(63, 33).

And, for the second, thus:

65. $CM\beta\beta$	(hypothesis)
66. $CM\gamma\gamma$	(hypothesis)
67. $CC\beta\gamma CM\beta\gamma$	(65)
68. $CC\beta\gamma CM\beta M\gamma$	(67, 24)
69. $CMC\beta\gamma MCM\beta M\gamma$	(68, RMC)
70. $CMC\beta\gamma CM\beta M\gamma$	(69, 52)
71. $CCM\beta M\gamma CM\beta\gamma$	(70, 66)
72. $CCM\beta M\gamma C\beta\gamma$	(71, 24)
73. $CMC\beta\gamma C\beta\gamma$	(70, 72).

From RM and the schema SM (proved in the last section) we immediately have the rule

$\vdash C\alpha\beta \rightarrow \vdash CSpCSq \ldots CM\alpha M\beta$, where p, q,...are all the variables in β that are not in α.

If we add the condition that all variables in β fall within the scope of an M or an S, we will have, for this β, $\vdash CM\beta\beta$ (by the lemma just proved), and so, from the above rule, the original RSM of the system Q.

6. *Predicate calculi based on Q and QK_t.* Although the informal motivation of the systems Q and QK_t has to do with the possibility of their propositions being of the form ϕx, the systems themselves, as developed above, are purely propositional calculi, with only propositional variables. We now consider the consequences of formally introducing individual-name variables and predicate variables, with quantifiers binding the former. It will be simplest to consider first what happens when we enlarge the modal system Q in this way.

For the quantifiers, postulates of the normal sort will suffice; for example, the two Łukasiewicz rules

$\Pi 1$: $\vdash C\alpha\beta \rightarrow \vdash C\Pi x\alpha\beta$
$\Pi 2$: $\vdash C\alpha\beta \rightarrow \vdash C\alpha\Pi x\beta$,

for x not free in α, with the definition of Σx as $N\Pi xN$. From these we obtain in the usual way the derived rules

$\Sigma 1$: $\vdash C\alpha\beta \rightarrow \vdash C\Sigma x\alpha\beta$, for x not free in β,
$\Sigma 2$: $\vdash C\alpha\beta \rightarrow \vdash C\alpha\Sigma x\beta$.

And substitution is modified in the presence of quantifiers in the usual ways.

It is necessary also to make additions to the rule RS1, and to the conditions in rules RS2 and RSM, which take care of the presence of other variables than propositional ones, and of the possible binding of some of these. The operator S, for permanent or necessary statability, attaches to propositions only, so that Sx and $S\phi$ would not be well formed, but we could express the permanent or necessary existence of x, and of any object that may be mentioned in the predicate ϕ (which might be something like '—is taller than y'), by prefixing S to formulae containing x or ϕ as their sole free variables. For x it would be simplest to introduce a constant two-place predicate I (for identity), and use $SIxx$, and for ϕ, $SI\Pi x\phi x$ would do. We then add to RS1

RS1′: $\vdash CS\alpha SI\Pi x\phi x$, where ϕ occurs in α,
RS1″: $\vdash CS\alpha SIxx$, where x occurs free in α,

and modify RS2 and RSM to

RS2′: $\vdash CSpCSq\ldots CSI\Pi x\phi xCSI\Pi x\psi x\ldots CSIxxCSIyy\ldots S\alpha$,
where $p, q,\ldots$, and $\phi, \psi,\ldots$, and $x, y\ldots$are all the free variables in α;

RSM′:

$\vdash C\alpha\beta \rightarrow \vdash CSpCSq\ldots CS\Pi x\phi xCS\Pi x\psi x\ldots CSIxxCSIyy\ldots CM\alpha\beta$,

where all the variables in β (bound or free) are within the scope of an M or an S, and where $p, q, \ldots, \phi, \psi, \ldots, x, y$, are all the variables which occur free in β but do not occur free in α.

As an example of a formula which is provable in quantified S5 but not in quantified Q we may take the 'Barcan formula' $CM\Sigma x\phi x\Sigma xM\phi x$, asserting that if it could be that something ϕ's then there is (actually) something that could ϕ, or that if it is the case at some time that something ϕ's then there is now something that ϕ's-at-some-time. (In a universe of non-permanent objects this is clearly an undesirable theorem). Even in S5 the following proof would not quite do:

74.	$C\phi xM\phi x$	$(CpMp)$
75.	$C\phi x\Sigma xM\phi x$	(74, $\Sigma 2x$)
76.	$C\Sigma x\phi x\Sigma xM\phi x$	(75, $\Sigma 1x$)
77.	$CM\Sigma x\phi x\Sigma xM\phi x$	(76, RCM).

For in the consequent of 76 there is a variable not within the scope of an M, even though it is only one in a quantifier, so that RCM is not applicable. However, a more indirect proof *is* possible in S5, and also in the weaker 'Brouwersche' system, which may be axiomatized by the rule RM of Section 4, with the axioms 23 (unrestricted), 24, and 35. For we have

78.	$CM\phi xM\phi x$	(p.c.)
79.	$CM\phi x\Sigma xM\phi x$	(78, $\Sigma 2x$)
80.	$CN\Sigma xM\phi xNM\phi x$	(79, p.c.)
81.	$CMN\Sigma xM\phi xMNM\phi x$	(80, RM, 23)
82.	$CNMNM\phi xNMN\Sigma xM\phi x$	(81, p.c.)
83.	$C\phi xNMN\Sigma xM\phi x$	(82; $CpNMNMp$, from 35)
84.	$C\Sigma x\phi xNMN\Sigma xM\phi x$	(83, $\Sigma 1x$)
85.	$CM\Sigma x\phi xMNMNx\Sigma M\phi x$	(84, RM, 23)
86.	$CM\Sigma x\phi x\Sigma xM\phi x$	(85; $CMNMNpp$, from 35).

In Q, the move from 80 to 81 is blocked by the restriction on 23. Or, to put it another way, with our axiomatization of S5 we would proceed from 80 to 81 via

80′ $CN\Sigma xM\phi xMNM\phi x$ (80, $CpMp$),

from which 81 follows by RCM—but not when RCM is restricted as in Q; in Q we could only get

81′ $CSIxxCMN\Sigma xM\phi xMNM\phi x$.

It is also necessary, when quantifiers binding individual variables are introduced into Q, to modify the rule of detachment to

$$\vdash\alpha,\ \vdash C\alpha\beta \rightarrow \vdash CSIxxCSIyy\ldots\beta,$$

where $x, y, \ldots$ are all the individual variables which occur free in α but not in β. The effects of this modification are not very severe; for example, it does not block the usual derivation of the rules $\Sigma 1$ and $\Sigma 2$ from the rules $\Pi 1$ and $\Pi 2$ and the definition of Σ. What it does block is the derivation of the theorem $\Sigma xIxx$, which asserts in effect that the universe is not empty. Even this is possibly not undesirable; there would in fact be no point in using free individual-name variables if the universe *were* empty, as there would then be no facts or falsehoods directly about individuals. But one might very well want to say that the universe *might have been* empty, or that it once was or eventually will be; i.e. one might not want to preclude the possibility expressed by $MN\Sigma xIxx$, and we would be taken to the contradictory of this by RM if we had $\Sigma xIxx$ as a theorem. To put it another way: a formula is a thesis of this system if it is not only not actually false, or not now false, but is not possibly false, or not false at any time, for any values of its variables. (This is the consideration that legitimates RM.) And Ixx is not false at any time (i.e. it is true whenever statable—true so long as x exists); the same is true of $CIxx\Sigma xIxx$; but perhaps $\Sigma xIxx$ (which has no free variables that could stand for individuals whose nonexistence would destroy its statability) was or will be or could be false. Hence the modification of detachment to block the derivation of the last from the former two. An alternative would be to admit as theorems all formulae of the system which are universally true *now*, and drop RM, or more accurately so modify RSM that RM is not derivable from it. But this is a type of system that has yet to be properly constructed.

The analogous modifications of QK_t are easy enough to work out. We need T1′, T1″, Y1′, and Y1″ analogous to RS1′ and RS1″; T2, Y2, FT1, and FY1 need to have added antecedents in $\mathcal{T}$ and $\mathcal{Y}$ analogous to the additions in S to RS2 and RSM; and detachment must be modified exactly as in Q.

7. *Further problems.* Tampering with the allegedly 'minimal' tense logic K_t, as we observed at the outset, is only legitimate if we refuse to regard that calculus as a mere by-product of definitions introduced into the first-order theory of the earlier-later relation between instants. And if we do refuse to see things in this way, it is natural to go further and see them, positively, in the opposite way, i.e. to treat the first-order theory of the earlier-later relation as a mere by-product of tense logic. If our tense logic is K_t, we obtain the old 'definitions' of P and F in the form of equivalences. In detail, we introduce special propositional variables a, b, c for which we have

A1. Ma, A2. $CMKapNMKaNp$, A3. Σaa;

and define the quasi-predication $(\alpha)a$ (where α is a proposition of tense logic) as $MKa\alpha$ and Uab as $(Fb)a$. The M used in these axioms and definitions is defined thus:

$$M^0\alpha = \alpha$$
$$M^{n+1}\alpha = APM^nFM^n\alpha$$
$$M\alpha = \Sigma n M^n\alpha$$

(in effect this equates Mp with 'p at some time'); and for this M, given K_t for P and F, we have S5, together with $CPpMp$ and $CFpMp$.

What kind of earlier-later logic do we get if we append these postulates, not to K_t, but to our new minimal tense logic QK_t? Do we, for instance, obtain the system Q for the M just defined? And is there a case for adding the postulate Sa, or other postulates involving S which concern 'instant-propositions'?

That is one set of problems which still faces us. Another is this: there are many alternative solutions to the problem of devising a tense logic that is suitable for non-permanent existents. We might, for example, drop individual name-variables altogether and just use *general* names a, b, c, with an undefined individuating propositional form $\epsilon'ab$, to be read as 'The only thing ever to be an a is a b'. For use with a 'name-logic' of this sort K_t seems unobjectionable; the complications can go into the name-logic. Given such an alternative, can QK be defined within it, or vice versa? If the answer is negative, either way, what further postulates might improve the situation?

A BIBLIOGRAPHY OF TENSE LOGIC

(ITEMS in the present collection are omitted. So are papers which are not strictly about tense logic, but merely about modal logic, or general philosophical reflections about time. I include, however, papers about Diodorean modal logic in which that is treated as definable within tense logic, and formal treatments of time and change, e.g. von Wright's, within which tenses are definable, though they are not taken as primitive.)

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Comments on Items 5, 45, *and* 46

I comment on 45 to make an acknowledgement. In the course of a fairly wide-ranging survey, it anticipates P. Kribs's set of possible 'states' formed by conjoining positive and negative Hamblin tenses (*Past, Present, and Future*, pp. 92–94), giving the number of such possible 'states' as sixty-two.

I comment on 5 and 46 because they contain new results that ought to be generally known. In 5, Bull shows that if we assume a set of postulates for linear time, infinite in both directions, with the earlier-later relation transitive, these postulates including a 'mirror-image rule', we may distinguish dense, continuous, and discrete time by subjoining the following further axioms (where $Lp = HGp = p$-always):

For density: $CGGpGp$
For continuity: $CGGpGp$, $CLCGpPGpCGpHp$
For discreteness: $CLCGppCGpHp$ (= $CHCGppCpCGpHp$)

Certain results from 46 were communicated by Kamp to a *UCLA* Logic Colloquium in February 1967. These results could, I think, be expressed as follows: if we write *pa* for 'It is the case at *a* that *p*' and '*Rab*' for '*a* is earlier than *b*', the truth-conditions for 'It will be that' and 'It has been that' are

$$(Fp)a = \Sigma bKRabpb$$
$$(Pp)a = \Sigma bKRbapb.$$

Kamp introduces two 2-place tenses which are symbolized in *Past, Present, and Future*, pp. 106 ff. by Φ and Ψ, but for which he now uses S (for 'since') and U (for 'until'), which have the truth-conditions

$$(Spq)a = \Sigma bKRbaKpb\Pi cCKRbcRcaqc$$
$$(Upq)a = \Sigma bKRabKpb\Pi cCKRacRcbqc,$$

i.e. *Spq* or '*q* since *p*' is true at *a*, say now, if *p* has been true and *q* is true at all moments between then and now, and *Upq*, or '*q* until *p*', analogously. These statements of truth-conditions have the general form

$$(f(p_1, p_2, \ldots, p_n))a = g(p_1, p_2, \ldots, p_n, a),$$

where *g* is definable within a first-order theory in which *R* is the only non-logical constant. We might define a 'tense' as any sentential operator *f* whose truth-conditions can be given in this way. The principal results communicated by Kamp can now be stated as follows:

(i) if we assume that the *R*-series is linear, dense, and infinite both ways, and that *R* is transitive, all 'tenses', in the sense just indicated, are definable in terms of *S*, *U*, and truth-functions.

(ii) *S* and *U* are not themselves definable in terms of (truth-functions and) any 1-place tenses, such as *F* and *P*.

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